# A NETWORK-BASED ALGORITHM FOR COMPUTING KEYNESIAN INCOME MULTIPLIERS IN MULTIREGIONAL SYSTEMS

**Dimitrios TSIOTAS**
Assistant Professor, Department of Regional and Economic Development, School of Applied Economic and Social Sciences, Agricultural University of Athens, Amfissa 33100, Greece
tsiotas@aua.gr

## Abstract

In the context of the Keynesian “multiplier effect” approach, regional economic growth and development are conceived as the result of changes in demand stimulating an iterative process of returns of income. Aiming to revisit this established regional economic model, promote multidisciplinary thinking, enjoy better supervision of computations and intuitive interpretation of the results, broaden the applicability of the model, and serve educational purposes in regional economics and development, this paper proposes an algorithm for computing Keynesian income multipliers in multiregional systems. Building on network connectivity, estimations of the regional shares of imports, marginal propensity to consume, and changes in demand, the proposed algorithm provides a framework for standardizing computations of the multiplier effect in multiregional systems. The algorithm is implemented in two theoretical scenarios, contributing to a deeper conceptualization of the computation of the Keynesian income multipliers, and an empirical case of the land interregional commuting network in Greece, providing insights into the developmental dynamics of the labor market (demand for employment) in Greece. Overall, the analysis highlights the symbiotic relationship between the multiplier effect and network structure in regional markets, promotes multidisciplinary thinking in regional science and economics, and provides a code of this network-based algorithm to motivate further research.

**Keywords:** regional markets, multiplier effect; export-base model; demand for employment; interregional commuting

**JEL classification:** R11, R15, R23, R41

## 1. Introduction

Economic growth and development are major concerns in the context of economic and social sciences (Karras, 2010; Acemoglu, 2012; De Dominicis, 2014; López-Bazo et al., 2014), as they are ruled by a high level of complexity (Kuznets, 1967; Tsiotas and Polyzos, 2018; Basile et al., 2021) and relate to the natural property of evolution, as well as to the immanent property of humanity to seek progress and welfare through optimum exploitation (or management) of the available resources (and production coefficients). Although the concepts of economic growth and development were diachronically drivers of human and social behavior (Polyzos, 2019; Polyzos and Tsiotas, 2020), their place in the economic agenda has become prominent during the last century (Acemoglu, 2012), when global, national, and regional economies faced major historical challenges and rapid transformations (Elhorst, 2010; Nijkamp, 2011; Rodrigue et al., 2013; Constantin, 2021; Trusova et al., 2021). A prime milestone in the economic history of the last century can undoubtedly be found in the year 1936, in the work of John Maynard Keynes titled “*The General Theory of Employment, Interest, and Money*” (Keynes, 1937), abbreviated as “*The General Theory*” and ever since has become a seminal work in macroeconomics and economics policy (Capello, 2016; Polyzos, 2019). Getting his inspiration from the Great Depression of the ‘30s, Keynes conceived that the main flaw (and consequently the determinant of recessions and depressions) of the recession in that period was the insufficient aggregate demand, contrary to the classical economic approaches interpreting that the supply suffices to produce intrinsic demand and thus to shift the supply-demand relationship (Zeenat Fouzia et al., 2020) into new states of equilibrium (Lagos, 2007; Polyzos, 2019). Within this context, Keynes proposed that any form of government spending can contribute to the increase of aggregate demand, which successively increases consumption, which in turn subsequently increases the demand, and so forth, stimulating, therefore, a circular (demand-consumption-demand-…) process contributing to the overall increase of income and GDP (Capello, 2016; Polyzos, 2019). For instance, changes in one of the components of aggregate demand (such as increases in local consumption, investments, public spending, or exports) generate an increase in income, which in turn increases the aggregate demand through the increase in consumption, stimulating successively further cycles of increase in demand and income, proportionally to the marginal propensity to consume (Capello, 2016). In other words, Keynes conceived that economic growth is indifferent to the structural setting and dynamics of supply (which is by default adaptable to the market requirements) and it rather depends on the increasing demand for goods (products and services), stimulating an income multiplier effect through the subsequent changes in the consumption and employment (Capello, 2016). This iterative economic process of the endless cycles of economic prosperity is known as the "multiplier effect" (Lagos, 2007; Hamm, 2010; Capello, 2016; Polyzos, 2019), which is expressed by a coefficient that is by definition greater than one (in the context that no intermediate or external diffusion of demand is applicable) and measures the change in output (income, GDP, etc.) caused by a unit change ($\Delta D=1$) in an aspect of the aggregate demand (Capello, 2016).

By interpreting demand as the engine of growth and development (Capello, 2016), the Keynesian multiplier effect has conceptually driven the first-generation theories of regional and economic development in the 1950s (Capello, 2016; Polyzos, 2019) and generally changed the road of the modern economic thinking. For instance, considerable influences of this Keynesian macroeconomics approach can be found in all versions of the export-base model of Charles Tiebout and Douglass North (North, 1955; Tiebout, 1956, 1960), which interpret regional development in terms of interregional trade transactions of a small economic system. Provided that assumptions of self-sufficiency and equivalence in production are acceptable for a spatial economic system, the export-base model conceives exports as the prime driver of aggregate demand (while intrinsic investments are considered as a minor driver) that stimulate the engine of regional development through the multiplier effects' process (Lagos, 2007; Capello, 2016; Polyzos, 2019). This is done by measuring the change in aggregate demand in the base industry of a regional economy and afterward estimating the income multiplier. As Capello (2016) notes, one method for computing regional multipliers through direct estimation of the local propensity to purchase goods was proposed by Archibald (1967). The method builds on aggregating the local shares of the national-known household consumption attributes (which are more likely to be locally purchased), for a sufficient period to generate a time series of local spending, on which the marginal propensity to consume (MPC) can be estimated when regressed. Another method for computing regional multipliers was proposed by K.J. Allen (Allen, 1969), according to which the regional multiplier is approximated by the inverse of a region's GDP diffusion quantities (leakages). Once four types of leakages from the multiplier effect on income are known (savings; interregional imports; imports from abroad; and direct and indirect taxes) and their shares of income are consequently computed, their inverse can yield the multiplier's value (Capello, 2016).

Also, considerable influences of the Keynesian multiplier can be found in the Harrod-Domar model (Harrod, 1939; Domar, 1946), which was developed to interpret the dynamics of regional economic systems by assuming that regional development depends on the imports originating from other regions. In particular, interregional imports are considered a major driver of the growth rates of a local economy, which are capable of setting the equilibrium growth conditions less restrictive and more easily sustainable, compared to the case of a national economy closed to foreign trade (Capello, 2016). In this context, while the export-base theory of Tiebout and North points out the importance of external demand as the engine of development, the Harrod–Domar model highlights that the developmental dynamics of a region may also be intrigued by investments (Alexiadis and Ladias, 2011; Polyzos, 2019; Mirzaei et al., 2021) originating from other regions. However, according to the Harrod–Domar model, when a region grows, at certain returns of scale, it undertakes the double risk of either exploding (due to the continuous growth) or meeting a point of recession (to reset its growth). This approach also builds on the engine of the multiplier effect to interpret regional growth, which however this time is stimulated by changes in demand due to the increase in investments due to external demand instead of consumption (Capello, 2016).

A strand in regional economic research that goes beyond the conceptualization of the Keynesian multiplier can be found in Leontief's Input-Output (IO) model (Leontief, 1966), which generally studies the interdependence of production sectors in an economy. The holder of the Nobel Prize in Economics for the development of this model, Wassily Leontief (1905-1999), conceived a systemic approach expressing intersectional transactions, between buyer and seller sectors (or industries) of an economic system, expressed by linear algebra equations (Miller and Blair 2009). By inverting the matrix of the technical coefficients, which expresses the amounts of commodities per unit commodity by each sector, we obtain the so-called "inverse Leontief matrix" (also called the "multiplier matrix") expressing the output by each sector produced by unit changes in the aggregate demand addressed to each sector. According to Capello (2016), "*whereas in export-base theory the Keynesian multiplier is synthesized into a single value, in the input-output analysis is disaggregated into an n×n set of multipliers relative to every sector or good demanded*". Within this context, Leontief's IO model allows estimating the outcome due to changes in demand in economic sectors, by inverting the matrix of technical coefficients (Polyzos, 2006; Polyzos and Sofios, 2008; Miller and Blair, 2009). Although Leontief's IO model is submitted to the limitation of constant returns in production and lack of technical progress (Capello, 2016), it promoted modern economic thinking.

In the last decade, the newly established discipline of network science (Barabasi, 2016) that uses the network paradigm (Tsiotas and Polyzos, 2018) to model systems of interaction into graphs (Barthelemy, 2011; Ladias, 2011; Tsiotas and Polyzos, 2018; Tsiotas and Ducruet, 2021), has contributed in the study of IO economic systems at the global and national level. In particular, by representing a square matrix configuration, an IO table can be seen as a complex network of inter-sectorial connectivity (Dominguez et al., 2021), where complex network analysis (Barthelemy, 2011; Fortunato, 2010; Tsiotas, 2019) may apply to unveil topological properties in the IO structures (Cerina et al., 2015; Dominguez et al., 2021; Costa et al., 2022). Despite its modern emergence, network analysis has already promoted the relevant research in IO economic systems, providing insights into the evolution over time of the worldwide IO model (Cerina et al., 2015; Rio-Chanona et al., 2017); the properties of the European production network (Giammetti et al., 2020); the Japanese IO economic structure (Dominguez and Mendez, 2019; Dominguez et al., 2021); the transmission mechanisms of domestic and foreign shocks across the Italian IO business system (Costa et al., 2022); the Greek IO economic structure (Garcia-Muniz and Ramos-Carvajal, 2015); and more. Within this framework, this paper gets its inspiration from (*i*) the potential to represent multiregional systems as network structures; and (*ii*) the already fruitful contribution of using the network paradigm in the IO economic systems research; and developing a network-based algorithm for computing Keynesian income multipliers in multiregional systems, as conceived in the context of the export-base (North, 1955; Tiebout, 1956, 1960), the Harrod-Domar (Harrod, 1939; Domar, 1946), and relevant models. Although relevant research on Keynesian income multipliers has gotten much ahead, and the IO-based computational approaches prevailed in the regional economics' literature, this paper can claim a place of contribution to the relevant literature as: (*i*) it revisits established (first generation) regional economic models, using a modern computational approach, thus promoting multidisciplinary thinking; (*ii*) it inherits the merit of the export-base theory to deal with Keynesian income multipliers in single values, thus allowing better supervision of computations and an intuitive interpretation of the results, compared to the IO-based models; (*iii*) applies to large multiregional systems composed of many regions, where corresponding IO approaches are counter-intuitive; (*iv*) by supporting supervision and intuitive interpretation, it also serves educational purposes in regional economics and development, where there is still a long way to go.

The remainder of this paper is structured as follows; Section 2 provides the essential and describes the steps of the proposed algorithm for computing Keynesian income multipliers in multiregional systems. Section 3 applies the proposed algorithm (*i*) to a pair of scenarios (examples) of multiregional systems and (*ii*) to an empirical case, of interregional commuting in Greece; and, finally, in Section 4 conclusions are given.

## 2. Methodology and Data

### 2.1. Terminology and notations

In the context of the export-base theory (North, 1955; Tiebout, 1956), when the sizes of the base sector and the total sector are known, we can obtain the value of the Keynesian income multiplier according to the formula (Capello, 2016; Polyzos, 2019):

$$m_{Y/X} = \frac{1}{1 - c + m} \tag{1}$$

where $m_{Y/X}$ is the income multiplier, c is the marginal propensity to consume, and $m$ is the marginal propensity to import. Equation (1) originates from the Keynesian formula of income $Y$ (Polyzos, 2019):

$$Y = C + I + G + \left(X - M\right) \tag{2}$$

where $C$ stands for consumption; $I$ for investments; $G$ for government spending; and ($X$–$M$) for the trade balance ($X$ exports, $M$ imports). In particular, we can get equation (1): (*i*) by considering the variables of consumption ($C$) and imports ($M$) as functions of income $C=c_o+cY$ and $M=m_o+mY$ (Polyzos, 2019), where $c_o$, $m_o$ are the autonomous consumption and imports and replacing their equivalents in equation (2); and (*ii*) by obtaining the partial derivative from the converted equation (2) in respect to the exports variable ($X$), namely $m_{Y/X} = \frac{\partial Y}{\partial X}$.

Although equation (1) can become much more complex whether assumptions of (*i*) intermediate demand ($X \rightarrow X–m_X X$); (*ii*) diffusion of demand abroad ($X \rightarrow X–m_X X–m_F(X–m_X X)$); (*iii*) tax payments ($T=t_o–tY$); (*iv*) and government expenses ($G=g_o–gY$); may apply in the model (Polyzos, 2019), the essence behind the computation of the Keynesian income multiplier is that equals to the slope of the variable submitted to changes in demand ($\Delta D \rightarrow \Delta X$). For instance, in the context of the export-base theory, changes in demand for imports (interpreted as consumption due to foreign production: $\Delta\overline{C}$) may be seen as a stimulus for the multiplier effect, according to the relation:

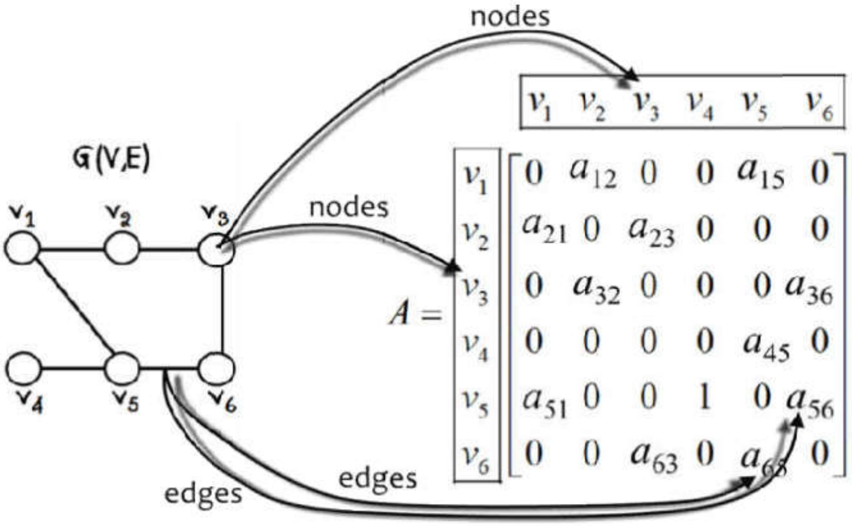
$$\Delta Y = m_{Y/X=\overline{C}} \cdot \Delta D = m_{Y/\overline{C}} \cdot \Delta\overline{C} \tag{3}$$

while, in the context of the Harrod-Domar theory, changes in demand for investments coming from abroad (Capello, 2016) ( $\Delta\overline{I}$ ), may suggest a stimulus for the multiplier effect, according to the relation:

$$\Delta Y = m_{Y/X=\overline{I}} \cdot \Delta D = m_{Y/\overline{I}} \cdot \Delta\overline{I} \tag{4}$$

Therefore, no matter how complex is, the Keynesian income multiplier is computed based on equation (1) and provides changes in income based on equations (3) and (4). However, when a multiregional system is considered, the concept of "abroad" (referring to a region's external environment) has a network structure, and thus changes captured in one region's demand are distributed throughout the other regions, by the structure of the multiregional system. This is the point where the network paradigm may contribute, toward standardizing the procedure of distributing such changes in demand (Polyzos, 2019), which currently lacks a network-based approach. Within this context, based on the network paradigm, a multiregional system can represent a network model (Tsiotas and Polyzos, 2018), which is a graph model (Barabasi, 2016; Barthelemy, 2011) defining a pair-set $G(V, E)$ of $n$ in number nodes ($V$) and $m$ in number edges/links ($E$). In algebraic terms, the simplest form of a graph model $G(V,E)$ is expressed by a (square) connectivity ($n \times n$) matrix of binary weights, which is called adjacency matrix $A=\{a_{ij}\}$. In an adjacency matrix, both rows and columns express nodes (Fig.1), whereas the matrix elements ($a_{ij}$) express the edges (the state of connectivity). In particular, when nodes (row=) $i$ and (column=) $j$ are connected, the corresponding element in the adjacency equals one ($a_{ij}$=1) and zero ($a_{ij}$=0) in cases of no connectivity.

**Fig.1. A graph $G(V,E)$ and its Adjacency Matrix $A=\{a_{ij}\}$**



By converting a multiregional system into a network model, further computations of distributing the changes in demand for computing the Keynesian income multipliers may apply. Such computations are easy to standardize by moving within the adjacency instead of drawing diverse paths. For doing so, the proposed algorithm is briefly described in the next section.

## 2.1. Description of the proposed network-based algorithm for computing Keynesian income multipliers in multiregional systems

The algorithm for computing Keynesian income multipliers in multiregional systems is described as follows:

```
Start
Part 1: INPUT arguments configuration
[Step#1] Configure the Adjacency matrix (n×n) of the multiregional system
[Step#2] Configure a vector (n×1) including the change(s) in demand ($\Delta D=D_1-D_o$) observed for one or more regions.
[Step#3] Configure a vector with the imports coefficients, namely with the proportions of demand that a prefecture satisfies through imports.
[Step#4] Configure the vector of Marginal Propensities to Consume (MPC), per region.
Part 2: COMPUTATIONS
[Step#5] Compute the complementary coefficients (of intrinsic satisfaction of demand), by subtracting the vector of the imports coefficients from one.
[Step#6] Find the regions that have no sellers and set their imports coefficients equal to 1.
[Step#7] Find the number of seller regions.
[Step#8] Compute the shares of imports per region, evenly distributed into their seller regions.
[Step#9] Calculate the changes in demand per region, by their trade connectivity and imports shares.
[Step#10] Compute the income multipliers according to the relation $m=1/(1-MPC)$.
[Step#11] Compute the income changes according to the relation $\Delta P=m\cdot\Delta D$
End
```

In particular, in the first part of the algorithm, the researcher configures the required arguments for inputs. In the first step (Step#1), we (Step#1) we create the Adjacency matrix of the interregional system, which represents a directed and connected network, as it is shown in Fig.1. Next (Step#2) the researcher configures a node vector $\Delta D=\{d_i\}$ that includes the change(s) in demand ($d_i=\Delta D_i(t_1)-\Delta D_i(t_o)$) observed for one or more regions $i=1, 2, \ldots, n$. This vector has a length of $n$ elements (equal to the number of nodes or regions), where a non-zero entry $d_i \neq 0$ in the $i$-th place of this vector expresses a change in demand for the region (node) $i$. Next (Step#3), the researcher configures a vector $I=\{m\}$ with the imports coefficients, which also has a length of $n$ and expresses the proportions of demand that a prefecture satisfies through imports. For instance, a non-zero entry $m_i \neq 0$ in the $i$-th place of this vector expresses that region $i$ satisfies $m_i \cdot 100\%$ of its demand through imports and $(1-m_i)\cdot 100\%$ through intrinsic production. Next (Step#4), the researcher configures the vector of Marginal Propensities to Consume (MPC), per region, based on the methods (Archibald, 1967; Allen, 1969) previously described (Capello, 2016). After configuring the input arguments, in Step#5 the algorithm computes the complementary coefficients of vector $I$, expressing the intrinsic satisfaction of demand, namely the proportions (shares) of demand covered per region by their intrinsic production ($(1-m_i)\cdot 100\%$). Next (Step#6), the algorithm finds the regions that have no sellers and sets their imports coefficients equal to 1. In the next step (Step#7), the algorithm finds the number of seller regions; to which, afterward (Step#7), it evenly distributes the imports' demand per region, according to the relation:

$$D_{ij} = \frac{m_j \cdot 100\%}{k(-)_j} \tag{5}$$

where $k(-)_j$ is the incoming node degree (number of seller regions) of region $j$. Here, possible future upgrades may include uneven distribution of the imports' demand. In the next step (Step#9), the algorithm computes the changes in demand per region, by their trade connectivity and imports shares. After converting the Adjacency matrix into a $n \times n$ demand distribution matrix, the algorithm computes (Step#10) the income multipliers according to equation (1). This step can also enjoy future upgrades. Finally, in the last step (Step#11) the algorithm computes the income changes according to equations (4) and (5). The implementation of the algorithm is shown, in detail, in the next section's examples.

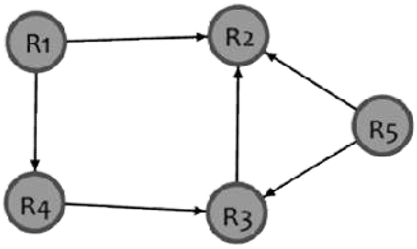
## 3. Results and Discussion

The proposed network-based algorithm is implemented on two examples, which are available in the textbook of Polyzos (2019) entitled *Regional Development* (language: Greek). The first example regards a five-region multiregional system experiencing changes in demand in one of its prefectures, while the second one regards a four-region multiregional system experiencing changes in demand in two of its prefectures. The step-by-step computational process of estimating the Keynesian income multipliers is shown in the following sub-sections. Although implemented for certain examples, the proposed algorithm is not restricted and applies to complex multiregional systems (composed of $n$ in number regions) enjoying changes in demand in multiple ($k \leq n$) regions. For implementation purposes and the sake of further research, the proposed algorithm is provided in a MATLAB *m*-function format, at the end of this paper (see Appendix).

### 3.1. Running the proposed algorithm for a five-region system, with changes in demand in one prefecture.

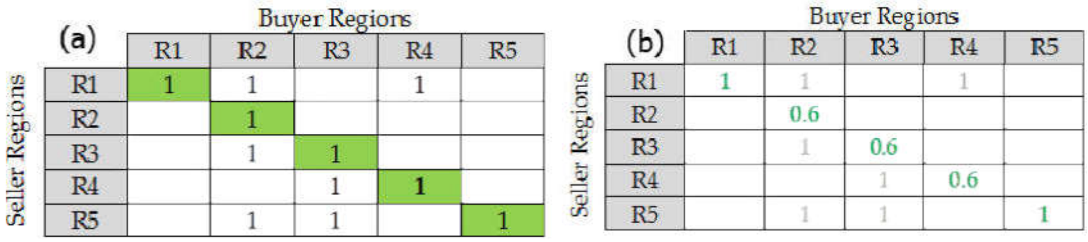
In the first example, the proposed algorithm runs for a five-region system, as it is shown in Fig.2. In this multiregional context, we assume that: (*i*) Region 3 ($R_3$) experiences a 10-unit change in its demand for investments ($\Delta I=10$); (*ii*) the marginal propensity to consume (MPC) in regions $R_1$ and $R_2$ is 60% ($MPC_{R_1,R_2} = 0.60$), while for the other regions ($R_3$, $R_4$, and $R_5$) is 70% ($MPC_{R_3:R_5} = 0.70$); and (γ) each region satisfies 40% of its demand through imports, equally distributed along its seller regions (and thus satisfies 60% of its demand from intrinsic production). Based on these assumptions, we calculate the multiplier effect on the regional income as follows:

**Fig.2. The five-region system of trade transactions, where the proposed algorithm applies**



First, we create the adjacency matrix of the five-node (five-region) multiregional system, according to the graph structure shown in Fig.3a. As it can be observed, the adjacency of the multiregional system configures a 5×5 matrix of a directed binary graph, where rows represent seller and columns buyer regions. When region $R_i$ is a seller of region $R_j$, we assign one in the concordant adjacency's element $a_{ij}$ =1. For instance, a non-zero value in cell $a_{32}$ =1 interprets that region $R_2$ is a buyer of region $R_3$, but the opposite is not true (thus suggesting that there is no edge $a_{23}$=0 in the adjacency matrix). Given that each region satisfies 40% of its demand through imports, we compute the complementary shares (100 – 40 =) 60%, representing the demand satisfied by a region's intrinsic production. These values are respectively assigned to the diagonal elements of the adjacency matrix, as it is shown in Fig.3b. As it can be observed, regions with no sellers have their diagonal elements equal to one (namely $a_{11}$=1 and $a_{55}$=1), since the "40-60" given rule is not applicable in their cases as all of their demand is satisfied through intrinsic production (no imports are applicable). By replacing the adjacency's diagonal elements with the shares of demand satisfied through intrinsic production, we start converting the adjacency matrix into a distribution matrix of demand coefficients throughout the network structure. Currently (Fig.3b), only the diagonal elements represent shares of demand and thus the process should move on to fill in the total matrix.

**Fig.3. (left) The adjacency matrix of the 5-node multiregional system of Fig.2 (colored diagonal elements) (right) The shares of demand satisfied by intrinsic production are assigned to the elements of the main diagonal**



(a)

| | | Buyer Regions | | | | |
|---|---|---|---|---|---|---|
| | | R1 | R2 | R3 | R4 | R5 |
| Seller Regions | R1 | 1 | 1 | | 1 | |
| | R2 | | 1 | | | |
| | R3 | | 1 | 1 | | |
| | R4 | | | 1 | 1 | |
| | R5 | | 1 | 1 | | 1 |

(b)

| | | Buyer Regions | | | | |
|---|---|---|---|---|---|---|
| | | R1 | R2 | R3 | R4 | R5 |
| Seller Regions | R1 | 1 | 1 | | 1 | |
| | R2 | | 0.6 | | | |
| | R3 | | 1 | 0.6 | | |
| | R4 | | | 1 | 0.6 | |
| | R5 | | 1 | 1 | | 1 |

In the next step, we equally distribute the shares of imports into the seller regions per node (region). To do so, for each buyer region $R_j$, we (column-wisely) divide the import shares (40%) to their incoming degree $k(-)$ (number of sellers), according to equation (5). For our example, the stepwise computational procedure is shown in Fig.4a:c and the results of the share of imports' distribution are shown in Fig.4d. In particular, we can read the results as follows: (*i*) region $R_1$ satisfies all of its demand through its intrinsic production; (*ii*) region $R_2$ satisfies 60% of its demand through its intrinsic production, whereas each of its sellers contributes to the region's demand at an amount of 13.33%; (*iii*) region $R_3$ satisfies 60% of its demand through its intrinsic production, whereas each of its sellers contributes to the region's demand at an amount of 20%; (*iv*) region $R_4$ satisfies 60% of its demand through its intrinsic production and 40% of its seller region ($R_1$); and (*v*) region $R_5$ satisfies all of its demand through its intrinsic production. If the computations are correct, the column-wise summands in the table of Fig.4d should yield ones expressing 1·100% satisfaction with a region's demand. The final matrix shown in Fig.4d represents the regional coefficients of shares in demand of our multiregional system.

**Fig.4. (a:c) The stepwise procedure of distributing a region's satisfaction of demand through imports into the seller regions; (d) The final coefficients matrix of shares in demand of the multiregional system**

(a)

| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | 1 | 1 | | 1 | |
| R2 | | 0.6 | | | |
| R3 | | 1 | 0.6 | | |
| R4 | | | 1 | 0.6 | |
| R5 | | 1 | 1 | | 1 |

(b)

| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | 1 | 1 | | 1 | |
| R2 | | 0.6 | | | |
| R3 | | 1 | 0.6 | | |
| R4 | | | 1 | 0.6 | |
| R5 | | 1 | 1 | | 1 |

(c)

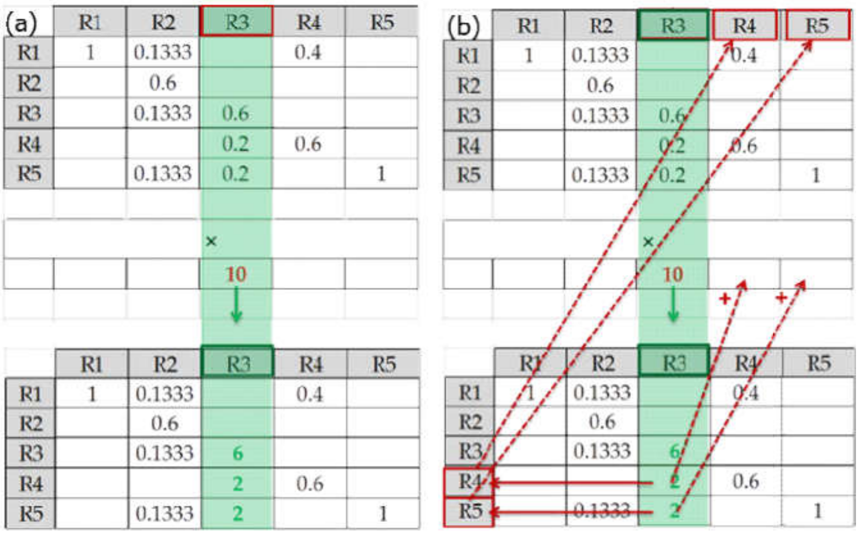
| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | 1 | 1 | | 1 | |
| R2 | | 0.6 | | | |
| R3 | | 1 | 0.6 | | |
| R4 | | | 1 | 0.6 | |
| R5 | | 1 | 1 | | 1 |

(d)

| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | 1 | 0.1333 | | 0.4 | |
| R2 | | 0.6 | | | |
| R3 | | 0.1333 | 0.6 | | |
| R4 | | | 0.2 | 0.6 | |
| R5 | | 0.1333 | 0.2 | | 1 |

By computing the matrix of shares in demand of our multiregional system (Fig.4d), we afterward distribute the change in demand experienced by region $R_3$ throughout the multiregional system. First, we start by distributing the 10-unit change in demand ($\Delta I$=10) experienced by region $R_3$ throughout its neighborhood. To do so, we multiply column 3 (corresponding to the buyer region $R_3$) with the 10-unit change in demand, as it is shown in Fig.5a. This multiplication yields a distribution of odds 6 ($R_3$): 2($R_4$): 2($R_5$), implying that 6 units from the 10-unit change in demand experienced by region $R_3$ are satisfied by the region's intrinsic production, whereas 2 (out of ten) units are satisfied through imports from region $R_4$ and another 2 units are satisfied through imports from region $R_5$. However, in the Keynesian multipliers context, the 2-unit changes in demand satisfied by regions $R_4$ and $R_5$ will cause secondary distribution of demand, according to the certain configuration of the multiregional network structure. This is because regions $R_4$ και $R_5$ are not closed, but open, economies, depending on other regions. Therefore, these 2-unit changes in demand are expected to initiate further rounds of computation. To facilitate the computations according to the proposed algorithm, we assign these 2-unit changes in demand to the respective places in the row with the initial changes in demand, as it is shown in Fig.5b.

**Fig.5. (a) The stepwise procedure of distributing the change in demand experienced by region $R_3$ throughout its neighborhood (b) According to the $R_3$ region's trade structure, secondary changes in demand should be also distributed to the sellers of region $R_3$**



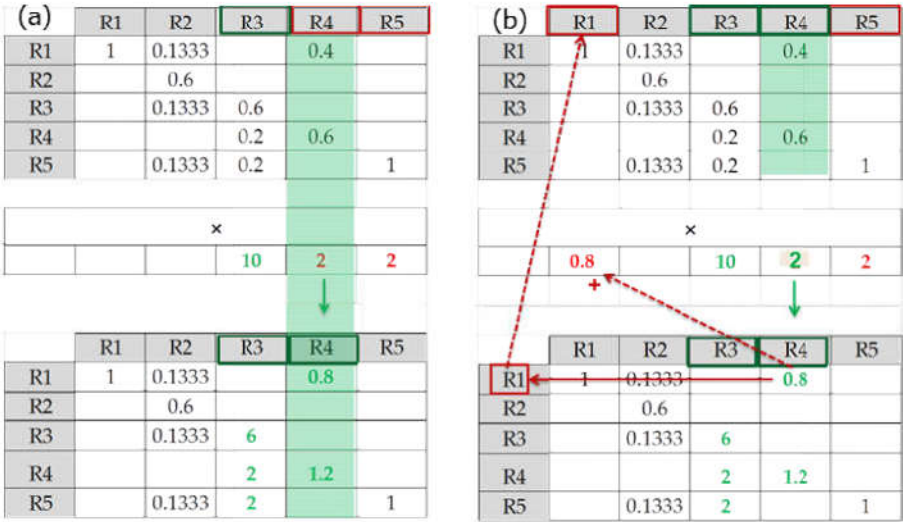
(a), (b)

| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | 1 | 0.1333 | | 0.4 | |
| R2 | | 0.6 | | | |
| R3 | | 0.1333 | 0.6 | | |
| R4 | | | 0.2 | 0.6 | |
| R5 | | 0.1333 | 0.2 | | 1 |

| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | 1 | 0.1333 | | 0.4 | |
| R2 | | 0.6 | | | |
| R3 | | 0.1333 | 6 | | |
| R4 | | | 2 | 0.6 | |
| R5 | | 0.1333 | 2 | | 1 |

To initiate the round of computing the secondary changes in demand, we multiply column 4 (this time corresponding to the buyer region $R_4$) with the 2-unit change in demand (that region $R_4$ experiences due to its trade connectivity with region $R_3$), as it is shown in Fig.6a. This multiplication yields a distribution of odds 0.8:1.2, implying that 0.8 units from the 2-unit change in demand experienced by region $R_4$ are satisfied by imports (originating from region $R_1$), whereas the other 1.2 units of demand are satisfied from the intrinsic production of region $R_4$ (Fig.6b).

**Fig.6. (a) The stepwise procedure of distributing the change in demand experienced by region $R_4$ throughout its neighborhood (b) The 0.8-unit secondary change in demand satisfied through imports from region $R_1$ initiates in turn another round of computations, this time for region $R_1$**



(a)

| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | 1 | 0.1333 | | 0.4 | |
| R2 | | 0.6 | | | |
| R3 | | 0.1333 | 0.6 | | |
| R4 | | | 0.2 | 0.6 | |
| R5 | | 0.1333 | 0.2 | | 1 |

×

| | | | | | |
|---|---|---|---|---|---|
| | | | 10 | 2 | 2 |

| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | 1 | 0.1333 | | 0.8 | |
| R2 | | 0.6 | | | |
| R3 | | 0.1333 | 6 | | |
| R4 | | | 2 | 1.2 | |
| R5 | | 0.1333 | 2 | | 1 |

(b)

| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | 1 | 0.1333 | | 0.4 | |
| R2 | | 0.6 | | | |
| R3 | | 0.1333 | 0.6 | | |
| R4 | | | 0.2 | 0.6 | |
| R5 | | 0.1333 | 0.2 | | 1 |

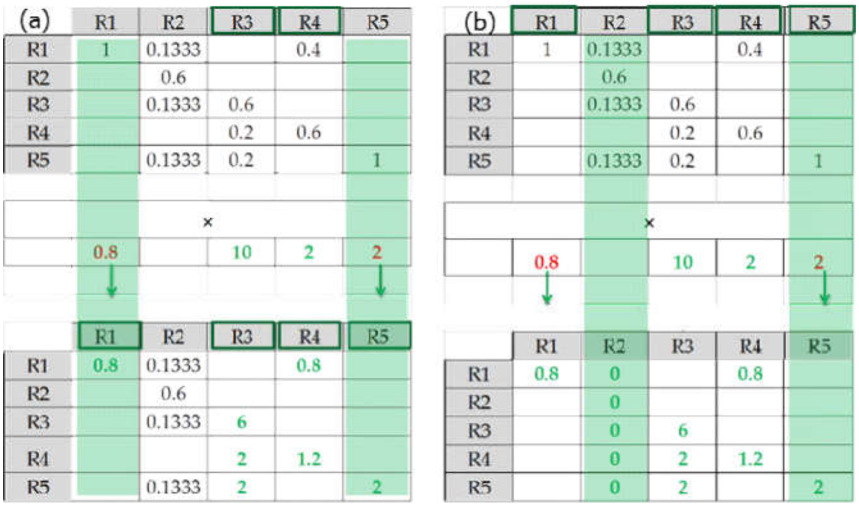
×

| | | | | | |
|---|---|---|---|---|---|
| | 0.8 | | 10 | 2 | 2 |

| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | ~~1~~ | ~~0.1333~~ | | 0.8 | |
| R2 | | 0.6 | | | |
| R3 | | 0.1333 | 6 | | |
| R4 | | | 2 | 1.2 | |
| R5 | | 0.1333 | 2 | | 1 |

Given that 0.8 units of the $R_4$ region's demand are satisfied through imports from region $R_1$, the algorithm in turn proceeds to another round of computing the distribution of secondary demands, this time for the case of region $R_1$ (Fig.7a). This step yields a trivial result since region $R_1$ is only a seller region, thus interpreting that region $R_1$ will satisfy all 0.8 units of change in demand through its intrinsic production. Similarly, as region $R_5$ is only a seller region, it will satisfy all its 2-unit change in demand through its intrinsic production (Fig.7a). Finally, as region $R_2$ is an only-buyer region, the initial 10-unit change in demand in region $R_3$ will not the demand of region $R_2$, thus we assign zeros at all entries of the demand distribution matrix (Fig.7b).

**Fig.7. (a) The stepwise procedure of distributing the secondary changes in demand into the only-seller regions $R_1$ and $R_5$ (b) Stepwise procedure of distributing the secondary changes in demand into the only-buyer region $R_2$, yielding a null distribution.**



(a)

| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | 1 | 0.1333 | | 0.4 | |
| R2 | | 0.6 | | | |
| R3 | | 0.1333 | 0.6 | | |
| R4 | | | 0.2 | 0.6 | |
| R5 | | 0.1333 | 0.2 | | 1 |

×

| | | | | | |
|---|---|---|---|---|---|
| | 0.8 | | 10 | 2 | 2 |

| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | 0.8 | 0.1333 | | 0.8 | |
| R2 | | 0.6 | | | |
| R3 | | 0.1333 | 6 | | |
| R4 | | | 2 | 1.2 | |
| R5 | | 0.1333 | 2 | | 2 |

(b)

| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | 1 | 0.1333 | | 0.4 | |
| R2 | | 0.6 | | | |
| R3 | | 0.1333 | 0.6 | | |
| R4 | | | 0.2 | 0.6 | |
| R5 | | 0.1333 | 0.2 | | 1 |

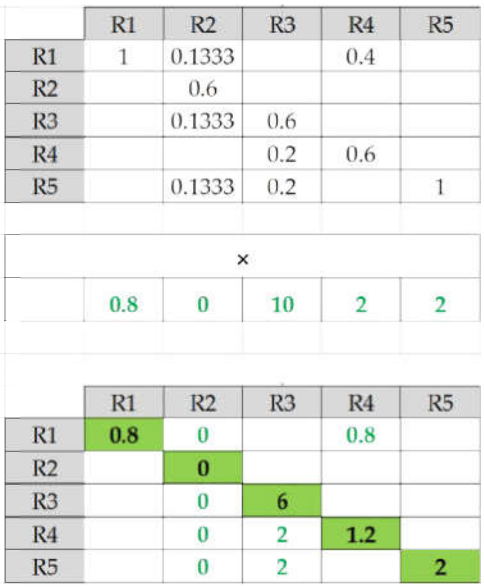
×

| | | | | | |
|---|---|---|---|---|---|
| | 0.8 | | 10 | 2 | 2 |

| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | 0.8 | 0 | | 0.8 | |
| R2 | | 0 | | | |
| R3 | | 0 | 6 | | |
| R4 | | 0 | 2 | 1.2 | |
| R5 | | 0 | 2 | | 2 |

Overall, based on the previous process, the demand distribution matrix is finalized and shown in Fig.8. As it can be observed from the main diagonal of the final demand distribution matrix, the initial 10-unit change in demand experienced by region $R_3$ is expected to cause a distribution of odds in demand throughout the multiregional system's regions as follows: 0.8 ($R_1$): 0 ($R_2$): 6 ($R_3$): 1.2 ($R_4$): 2 ($R_5$). These values are extracted from the main diagonal and should sum to the initial change in demand (0.8+0+6+1.2+2=10) if all computations were done correctly.

**Fig.8. The final conversion of the demand shares matrix to the demand distribution matrix**



| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | 1 | 0.1333 | | 0.4 | |
| R2 | | 0.6 | | | |
| R3 | | 0.1333 | 0.6 | | |
| R4 | | | 0.2 | 0.6 | |
| R5 | | 0.1333 | 0.2 | | 1 |

×

| | | | | | |
|---|---|---|---|---|---|
| | 0.8 | 0 | 10 | 2 | 2 |

| | R1 | R2 | R3 | R4 | R5 |
|---|---|---|---|---|---|
| R1 | 0.8 | 0 | | 0.8 | |
| R2 | | 0 | | | |
| R3 | | 0 | 6 | | |
| R4 | | 0 | 2 | 1.2 | |
| R5 | | 0 | 2 | | 2 |

In the final step of the algorithm, we calculate the expected changes in the regional income due to the Keynesian multiplier effect, according to equations (1) and (3), (4). After replacements and computations, we construct Table 1 with the final results.

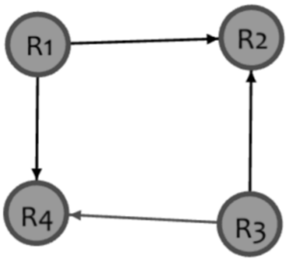
**Table 1. Final results of the Keynesian regional income multipliers for the 5-node multiregional system**

| Region | Change in Demand | MPC | m | Change in Income |
|---|---|---|---|---|
| $R_1$ | 0.80 | 0.60 | 2.50 | 2.00 |
| $R_2$ | 0.00 | 0.60 | 2.50 | 0.00 |
| $R_3$ | 6.00 | 0.70 | 3.33 | 19.98 |
| $R_4$ | 1.20 | 0.70 | 3.33 | 3.99 |
| $R_5$ | 2.00 | 0.70 | 3.33 | 6.66 |
| **Total** | 10.00 | | | 32.63 |

### 3.2. Running the algorithm to a four-region system, with changes in demand in two prefectures

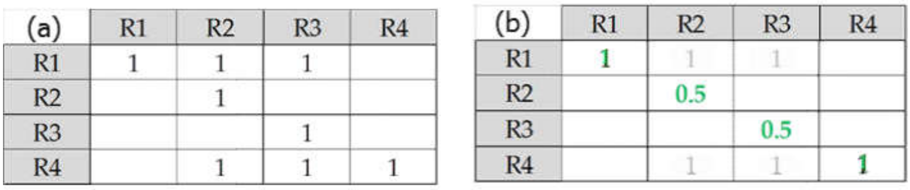
In the second example, the proposed algorithm runs for a four-region system, as it is shown in Fig.9. In this multiregional context, we assume that: (*i*) Region $R_2$ experiences a 10-unit change in its demand for investments ($\Delta I_2$=10), while region $R_3$ experiences a 20-units change in its demand for investments ($\Delta I_3$=20); (*ii*) the MPC in regions $R_1$ and $R_2$ is 60% ( $MPC_{R_1,R_2} = 0.60$ ), while in regions $R_3$ and $R_4$ is 70% ( $MPC_{R_3,R_4} = 0.70$ ); and (γ) each region satisfies 50% of its demand through imports, equally distributed along its seller regions (and thus satisfies 50% of its demand from intrinsic production). Based on these assumptions, we calculate the multiplier effect on the regional income as follows:

**Fig.9. The four-region multiregional system of trade transactions, where the proposed algorithm applies.**



First, we create the adjacency matrix of the 4-node (4-region) multiregional system, according to the graph structure shown in Fig.10a. The adjacency of the multiregional system configures a 4×4 matrix of a directed binary graph. Given that each region satisfies 50% of its demand through imports, we compute the complementary shares (100 – 50 =) 50%, which represent the demand satisfied by intrinsic production. We afterward assign these values to the diagonal elements of the adjacency matrix, as it is shown in Fig.10b. In this case, regions $R_1$ and $R_4$ are only-seller regions (and thus they have their diagonal elements equal to one), whereas $R_2$ and $R_3$ are only-buyer regions. Therefore, we start converting the adjacency matrix into a distribution matrix of demand coefficients throughout the network structure. As it can be observed, we obtain a distribution of odds in the shares of intrinsic demand satisfaction, as follows: 1 ($R_1$): 0.5 ($R_2$): 0.5 ($R_3$): 1 ($R_4$).

**Fig.10. (left) The adjacency matrix of the 4-node multiregional system of Fig.9 (right) The shares of demand satisfied by intrinsic production are assigned to the elements of the main diagonal**

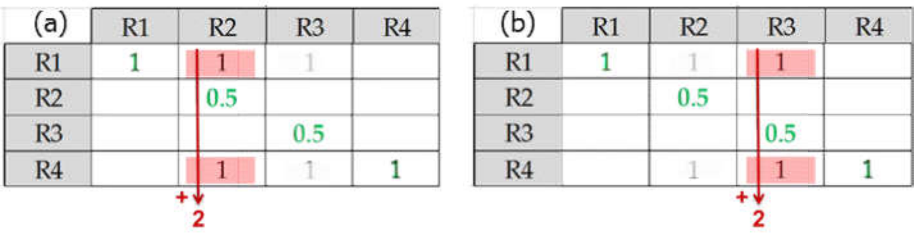


| (a) | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| R1 | 1 | 1 | 1 | |
| R2 | | 1 | | |
| R3 | | | 1 | |
| R4 | | 1 | 1 | 1 |

| (b) | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| R1 | 1 | 1 | 1 | |
| R2 | | 0.5 | | |
| R3 | | | 0.5 | |
| R4 | | 1 | 1 | 1 |

In the next step, we equally distribute the shares of imports into the seller regions per region, as it is shown in Fig.11. As can be observed in columns 2 and 3, regions $R_2$ and $R_3$ have the same two sellers, thus they equivalently (25%-25%) satisfy their demand through imports amongst their sellers.

**Fig.11. (a,b) The stepwise procedure of distributing a region's satisfaction of demand through imports into the seller regions**

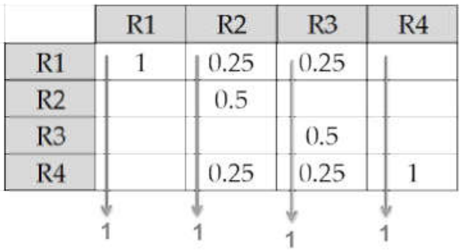


| (a) | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| R1 | 1 | 1 | 1 | |
| R2 | | 0.5 | | |
| R3 | | | 0.5 | |
| R4 | | 1 | 1 | 1 |

| (b) | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| R1 | 1 | 1 | 1 | |
| R2 | | 0.5 | | |
| R3 | | | 0.5 | |
| R4 | | 1 | 1 | 1 |

The results of the demand share distributions are shown in Fig.12, where we can read: (*i*) regions $R_1$ and $R_4$ satisfy all of their demand through their intrinsic production; and (*ii*) regions $R_2$ and $R_1$ satisfies 50% of their demand through its intrinsic production, whereas each of its sellers contributes to the region's demand at an amount of 25%. If the computations are correct, the column-wise summands in the table of Fig.12 should yield ones expressing 1·100% satisfaction of a region's demand.

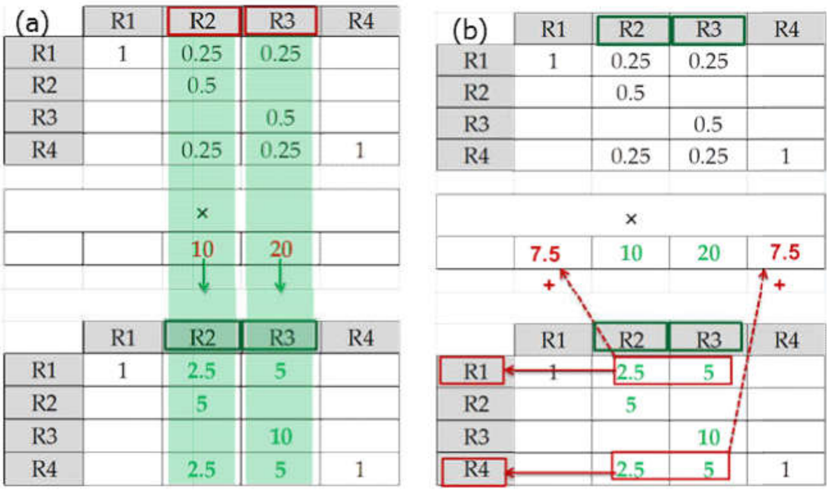
**Fig.12. The final coefficients matrix of shares in demand of the multiregional system**



| | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| R1 | 1 | 0.25 | 0.25 | |
| R2 | | 0.5 | | |
| R3 | | | 0.5 | |
| R4 | | 0.25 | 0.25 | 1 |

By computing the matrix of shares in demand of our multiregional system (Fig.4d), we afterward distribute the change in demand experienced by region $R_3$ throughout the multiregional system. First, we start by distributing the 10-unit change in demand ($\Delta I_2=10$) experienced by region $R_2$ throughout its neighborhood, along with the 20-unit change in demand ($\Delta I_3=20$) experienced by region $R_3$ throughout its neighborhood. To do so, we multiply column 2 (corresponding to the buyer region $R_2$) with the 10-unit change in demand, and column 3 (corresponding to the buyer region $R_3$) with the 20-unit change in demand, as it is shown in Fig.13a. For region $R_2$, this multiplication yields a distribution of odds 2.5($R_1$): 5($R_2$): 2.5($R_4$), implying that 5 units from the 10-unit change in demand experienced by region $R_2$ are satisfied by the region's intrinsic production, whereas 2.5 (out of 10) units are satisfied through imports from regions $R_1$ and $R_4$ respectively. For region $R_3$, this multiplication yields a distribution of odds 5($R_1$): 10($R_2$): 5($R_4$), implying that 5 units from the 20-unit change in demand experienced by region $R_3$ are satisfied by the region's intrinsic production, whereas 5 (out of 20) units are satisfied through imports from regions $R_1$ and $R_4$ respectively. Since region $R_2$ imports 2.5 units from region $R_1$ to satisfy its change in demand and region $R_2$ imports 5 units from region $R_1$ to satisfy its concordant change in demand, the overall secondary change in demand for region $R_1$ is the summand 2.5+5=7.5 units. Similarly, the overall secondary change in demand for region $R_4$ is also 2.5+5=7.5 units, as it is shown in Fig.13b.

**Fig.13. (a) The stepwise procedure of distributing the change in demand experienced by regions $R_2$ and $R_3$ throughout their neighborhoods (b) According to the multiregional trade network structure, secondary changes in demand should be also distributed to the sellers of regions $R_2$ and $R_3$**



| (a) | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| R1 | 1 | 0.25 | 0.25 | |
| R2 | | 0.5 | | |
| R3 | | | 0.5 | |
| R4 | | 0.25 | 0.25 | 1 |

×

| | | | | |
|---|---|---|---|---|
| | | 10 | 20 | |

| | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| R1 | 1 | 2.5 | 5 | |
| R2 | | 5 | | |
| R3 | | | 10 | |
| R4 | | 2.5 | 5 | 1 |

| (b) | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| R1 | 1 | 0.25 | 0.25 | |
| R2 | | 0.5 | | |
| R3 | | | 0.5 | |
| R4 | | 0.25 | 0.25 | 1 |

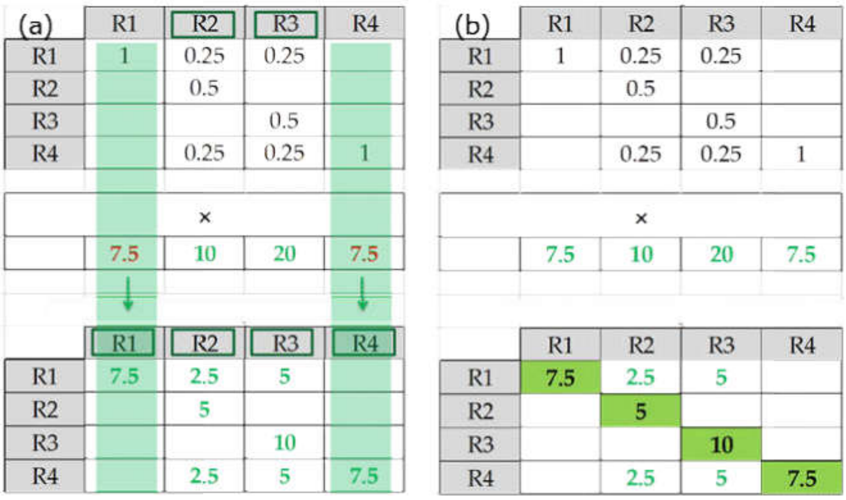
×

| | | | | |
|---|---|---|---|---|
| | 7.5 | 10 | 20 | 7.5 |

| | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| R1 | 1 | 2.5 | 5 | |
| R2 | | 5 | | |
| R3 | | | 10 | |
| R4 | | 2.5 | 5 | 1 |

Therefore, we proceed and calculate how these 7.5-unit secondary changes in demand are distributed throughout the neighbors of regions $R_1$ and $R_4$ respectively, as it is shown in Fig.14a,b. As regions $R_1$ and $R_4$ are seller-only regions, these changes in demand are trivial and will be satisfied by the intrinsic production of these regions. Overall, based on the previous process, the demand distribution matrix is finalized as it is shown in Fig.14b. As it can be observed from the main diagonal of the final demand distribution matrix, the initial 10-unit change in demand experienced by region $R_2$ and the 10-unit change in demand experienced by region $R_3$ are expected to cause a distribution of odds in demand throughout the multiregional system's regions as follows: 7.5 ($R_1$): 5 ($R_2$): 10 ($R_3$): 7.5 ($R_4$). These values are extracted from the main diagonal and should sum to the initial change in demand (7.5+5+10+7.5=(10+20=)30) if all computations were done correctly.

**Fig.14. (a) The stepwise procedure of distributing the changes in demand experienced by regions $R_1$ and $R_4$ throughout their neighborhoods (b) The 7.5-unit secondary changes in demand satisfied through imports from regions $R_1$ and $R_4$ initiates a secondary round of computations, this time for regions $R_1$ and $R_4$**



| (a) | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| R1 | 1 | 0.25 | 0.25 | |
| R2 | | 0.5 | | |
| R3 | | | 0.5 | |
| R4 | | 0.25 | 0.25 | 1 |

×

| | | | | |
|---|---|---|---|---|
| | 7.5 | 10 | 20 | 7.5 |

| | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| R1 | 7.5 | 2.5 | 5 | |
| R2 | | 5 | | |
| R3 | | | 10 | |
| R4 | | 2.5 | 5 | 7.5 |

| (b) | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| R1 | 1 | 0.25 | 0.25 | |
| R2 | | 0.5 | | |
| R3 | | | 0.5 | |
| R4 | | 0.25 | 0.25 | 1 |

×

| | | | | |
|---|---|---|---|---|
| | 7.5 | 10 | 20 | 7.5 |

| | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| R1 | 7.5 | 2.5 | 5 | |
| R2 | | 5 | | |
| R3 | | | 10 | |
| R4 | | 2.5 | 5 | 7.5 |

In the final step of the algorithm, we calculate the expected changes in the regional income due to the Keynesian multiplier effect, according to equations (1) and (3), (4). After replacements and computations, we construct Table 2 with the final results.

**Table 2. Final results of the Keynesian regional income multipliers for the 4-node multiregional system**

| Region | Change in Demand | MPC | m | Change in Income |
|---|---|---|---|---|
| $R_1$ | 7.50 | 0.60 | 2.50 | 18.75 |
| $R_2$ | 5.00 | 0.60 | 2.50 | 12.50 |
| $R_3$ | 10.00 | 0.70 | 3.33 | 33.30 |
| $R_4$ | 7.50 | 0.70 | 3.33 | 24.98 |
| **Total** | 30.00 | | | 89.53 |

### 3.3. Estimating the Keynesian income multipliers based on the interregional commuting network in Greece

In the final step of the analysis, we compute the Keynesian income multipliers for the land interregional commuting network in Greece (LGCN), based on the proposed network-based algorithm. The LGCN is a one-layer graph model $G(39;121)$ with commuting weights (Fig.15), composed of (*i*) $n$=39 nodes expressing the land (non-insular) capital cities of the Greek prefectures and (*ii*) $m$=121 edges expressing the number of commuters moving daily from a node (city) $i$=1,2,…,39 to node $j$=1,2,…,39 ($i \neq j$) for work. Nodes in the LGCN are geo-referenced at the coordinates of the city centers (WGS Web Mercator). Data for the construction of the node-set were extracted from the Google Digital Mapping Services (Google Maps, 2019), whereas commuting data concern employed persons with residence in the area by place of work, extracted from the 2011 national census of Greece (Hellenic Statistical Service – ELSTAT, 2011). To set computations of the Keynesian income multipliers for the LGCN reasonable, we conceptualize commuting as an aspect of labor demand in the interregional market of Greece (Polyzos, 2019; Tsiotas and Polyzos, 2021; Tsiotas et al., 2018, 2022). In particular, we can conceive the number of incoming commuters as an aspect of imports satisfying the labor demand in a regional economy, and inversely the number of incoming commuters as an expression of labor exports. Within this framework, we can approximate a region's share of labor imports (imp$C_i$) by the ratio of the number of incoming commuters to the employed labor force, according to the formula:

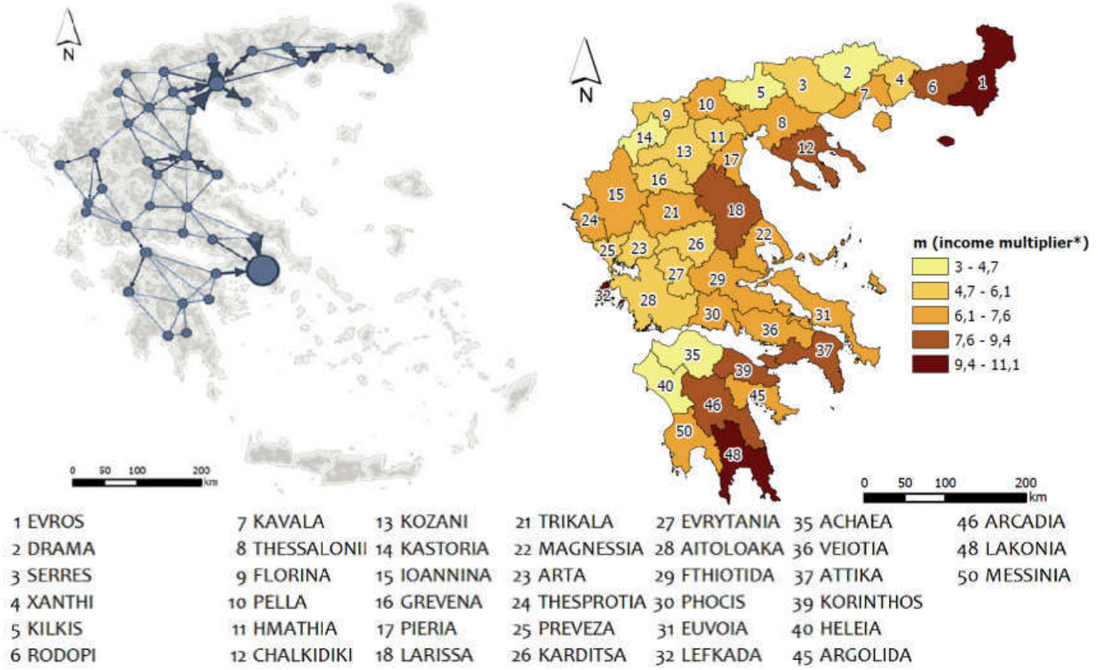
$$\text{imp}C_i = \frac{s_i(-)}{L_i - L_i^{unemp}} = \frac{s_i(-)}{L_i^{emp}} \qquad \textbf{(6)}$$

where $s_i(+)$ is the number of incoming commuters of region $i$; $L_i$ is a region's total labor force, computed by the sum of the employed $L_i^{emp}$ and unemployed $L_i^{unemp}$ labor force. Further, we can approximate a region's marginal propensity to consume "employment" or "labor" (MPC) by the ratio of the employed to the total labor force, according to the formula:

$$MPC_i = \frac{L_i^{emp}}{L_i} = \frac{L_i - L_i^{unemp}}{L_i} \qquad \textbf{(7)}$$

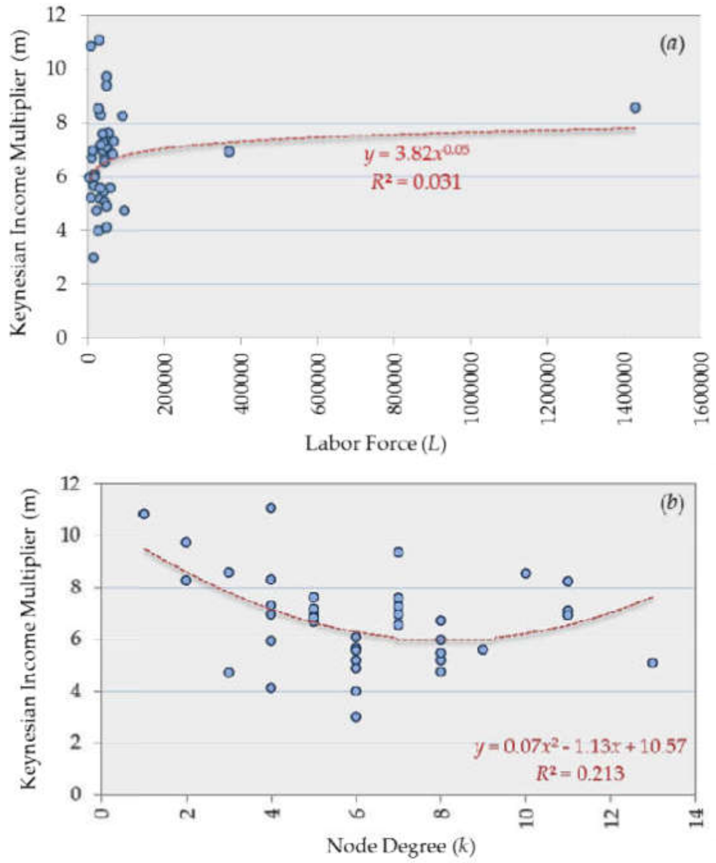
Based on the previous assumptions and after configuring the connectivity (commuting weights) matrix of the LGCN, we can compute the Keynesian income multipliers by applying the proposed algorithm (its code is available in an *m*-function format in the Appendix). Here, instead of using the adjacency matrix, we use the weights connectivity matrix of the LGCN, as the proposed algorithm is written to operate over the trade balances, namely to keep only one link (this with the positive difference $w_{ij}$–$w_{ji}$) between two network nodes $i,j$. Within this context, we estimate the Keynesian income multipliers for the LGCN and the results are shown in Fig.15. As it can be observed, in the context of the Greek market of labor demand, regions with the highest income multipliers are allocated along an "S"-type direction, which coincides with the major pattern describing regional development over time in Greece (Polyzos, 2019; Tsiotas, 2021). The cases with the highest income multipliers are the edge regions (Polyzos, 2015; 2019) Evros (1) and Lakonia (48), an observation that highlights the vitality of employment in edge regions and the importance of regional policies supporting the periphery (Xanthos et al., 2012; Alexiadis et al., 2013; Goula et al., 2015; Tsiotas and Tselios, 2022), addressing avenues of further research. Further, amongst the highest income multipliers we can find: (*i*) the metropolitan prefecture of Attika (37); (*ii*) Larissa (18), which is a region of great intermediacy in the land transport network in Greece (Tsiotas, 2021); and (*iii*) Chalkidiki (12), which is a neighbor and tourism destination of the metropolitan region of Thessaloniki (8).

**Fig.15. (left) The land interregional commuting network in Greece (LGCN) (right) Spatial distribution of employment-based* income multipliers computed on the LGCN**



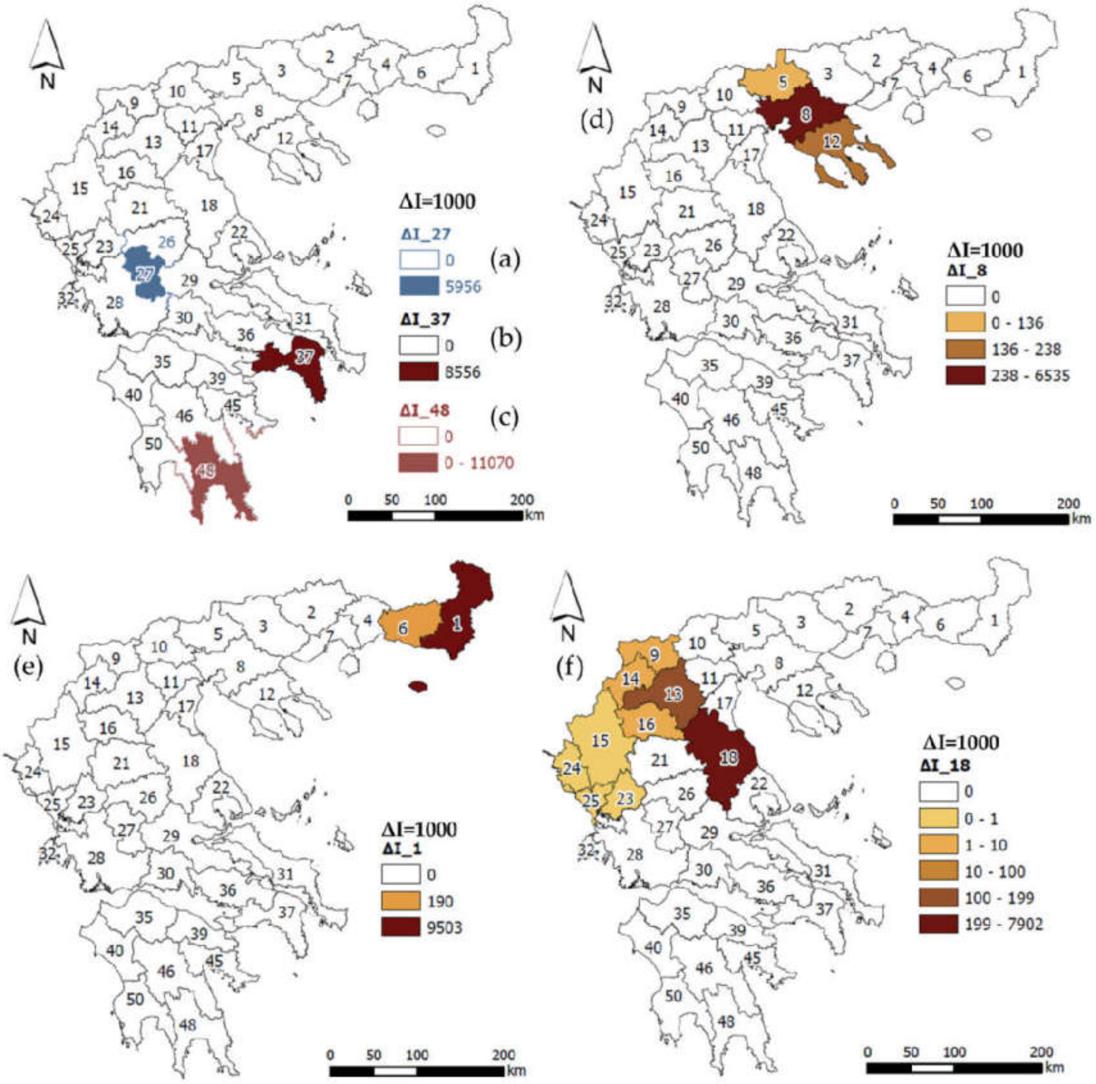
To further examine the implementation results of the proposed algorithm, we construct the scatter plots shown in Fig.16. The first scatter plot illustrates the correlation between the labor force of the LGCN (horizontal axis) and the Keynesian income multipliers (vertical axis), and the second scatter plot the correlation between the LGCN's node degree (horizontal axis) and the income multipliers (vertical axis) computed on the LGCN's demand for employment. As it can be observed, the income multipliers appear uncorrelated to the labor market size (Fig.16a), whereas there are indications that they correlate (with 8.8% 2-tailed significance) to the network structure (connectivity). Moreover, the "U"-shaped correlation pattern, implies that nodes of medium connectivity tend on average to be subject to lower multiplier effects than both highly and not-highly connected nodes. This observation impressively interprets that either the states of isolation or high connectivity are network structures stimulating higher multiplier effects. Of course, these indications address avenues of further research on the relationship between network structure and the Keynesian multiplier effect and should be studied in more detail.

**Fig.16. Scatter plots illustrating the correlation between the labor force of the LGCN and the Keynesian income multipliers (computed on the LGCN's demand for employment) and (b) the network degree of the LGCN and the Keynesian income multipliers (computed on the LGCN's demand for employment)**



Finally, we examine some scenarios of change in labor demand in the prefectures of Athens (37), Thessaloniki (12), Eurytania (27), Evros (1), Lakonia (48), and Larissa (18). Regions (37) and (37) are metropolitan in the Greek labor market; Evros (1) and Lakonia (48) are edge regions, as it is observed in Fig.15; Eurytania (27) is the least populated region in Greece, and Larissa (18) is a central region in the LGCN (Fig.15). By assuming, in all cases, a change in demand for employment equal to ΔI=1000 workers, the results of the multiplier effect are shown in Fig.17. As it can be observed, for the prefectures Eurytania (27) (Fig.17a), Athens (37) (Fig.17b), and Lakonia (48) (Fig.17c), a ΔI=1000 change in demand for employment will cause an isolated multiplier effect applicable to each prefecture. On the other hand, for the cases of Thessaloniki (8) (Fig.17d), Evros (1) (Fig.17e), and Larissa (18) (Fig.17f) a ΔI=1000 change in demand for employment will also cause spillover (Giovanni and Francesco, 2008; Andersson, 2012) multiplier effects, which for the prefecture of Larissa (18) are more extensive (spread throughout 8 prefectures).

**Fig.17. Results of implementing the proposed algorithm on six scenarios of change in labor demand ΔI=1000 in the prefectures of (a) Eurytania (27); (b) Athens (37); (c) Lakonia (48); (d) Thessaloniki (8); (e) Evros (1); and (f) Larissa (18)**



## 4. Conclusions

The Keynesian macroeconomics approach of the “multiplier effect” influenced the way the 1st generation theories in regional science have perceived and described the engine of regional economic growth and regional development. In the context of this approach, regional economic growth and development are conceived as the result of changes in demand stimulating an iterative process of returns of income. Despite the constraints due to modeling assumptions, in epistemological terms, the evolution of theories and models building on the Keynesian multiplier effect has proven that this approach has been subjected to a multiplier effect itself, as it stimulated succeeding theories and perceptions (e.g. the export-base model of Tiebout and North, the Harrod-Domar’s model, the Leontief’s IO model, and many other similar approaches), enjoyed voluminous and multidisciplinary research and promoted the way of economic thinking. Today, in the era of complexity and connectivity ruling all aspects of scientific research and life, economic growth and development are revisited in the context of the network paradigm supporting the description of complex systems. By getting its inspiration from its multidisciplinary-friendly configuration, the potential to represent multiregional systems as network structures, and the fruitful contribution of network science in the IO economic systems research, this paper developed a network-based algorithm for computing Keynesian income multipliers in multiregional systems. Aiming to (*i*) revisit an established regional economic model through a modern computational approach; (*ii*) promote multidisciplinary thinking; (*iii*) retrieve the Keynesian income multipliers' benefits of better supervision of computations, mono-variable (one good) consideration, and intuitive interpretation of the results; (*iv*) broaden the applicability of the model; and also (*v*) serve educational purposes in regional economics and development; the proposed algorithm built on inputs of a network connectivity (adjacency) matrix and estimations of the regional shares of imports, marginal propensity to consume, and changes in demand, and provided a framework for standardizing computations of the multiplier effect in multiregional systems, regardless their size and the level of their complexity. The algorithm consists of steps distributing the shares and volumes in demand throughout the adjacency (connectivity) matrix of the multiregional system and afterward estimating the multiplier coefficients and the changes in income per region (node). After its description, the proposed network-based algorithm was implemented in two theoretical scenarios, which are available in the textbook Polyzos (2019), and in an empirical case of the land interregional commuting network in Greece, where nodes represented land (non-insular) capital cities of the Greek prefectures and the edges the number of commuters moving between regions for work. The theoretical scenarios contributed to a deeper conceptualization of the computation of the Keynesian income multipliers, whereas the empirical scenario revealed patterns providing insights into the developmental dynamics of the labor market (demand for employment) in Greece. In particular, the analysis revealed that regions with the highest income multipliers are allocated along an "S"-type direction, coinciding with the major pattern of regional development in Greece, and edge regions enjoy the highest regional multipliers. Also, it revealed a "U"-shaped correlation pattern between the node degree and the income multipliers, implying that either the states of isolation or high connectivity are network structures stimulating higher multiplier effects. Finally, this paper examined scenarios of 1000-unit change in labor demand in the Greek commuting network and revealed that network intermediacy and less the regional population is a stimulus for spillover multiplier effects. Overall, the analysis revealed the symbiotic relationship between the multiplier effect and network structure in regional markets, promoted multidisciplinary thinking in regional science and economics, and provided a network-based algorithm for computing Keynesian income multipliers to motivate further research.

## 5. References

AUTHORREFERENCE

Alexiadis, S., Ladias, C., Hasanagas, N., (2013) "A regional perspective of the Common Agricultural Policy", *Land Use Policy*, 30(1), pp.665-669.

Alexiadis, S., Ladias, Christos, Ap., (2011) "Optimal allocation of investment and regional disparities", *Regional Science Inquiry*, 3(2), pp.45-59.

Allen K. J., (1969) "The Regional Multiplier: Some Problems in Estimation", in J.B. Cullingworth and S.C. Orr (Eds), *Regional and Urban Studies: a Social Science Approach*, Allen & Unwin, London, pp. 80–96.

Andersson, M., Weiss, J. F., (2012) "External trade and internal geography: Local export spillovers by industry characteristics and firm size", *Spatial Economic Analysis*, 7(4), pp.421-446.

Archibald G. (1967), 'Regional Multiplier Effects in the United Kingdom', *Oxford Economic Papers*, vol. 19, no. 1, pp. 22–45.

Archibald, G. C., (1967) "Regional multiplier effects in the UK", *Oxford Economic Papers*, 19(1), pp.22-45.

Barabasi, A.-L., (2016) Network science, Cambridge: Cambridge University Press.

Barthelemy, M., (2011) "Spatial networks", *Physics Reports*, 499, pp.1–101.

Basile, R., Commendatore, P., Kubin, I., (2021) "Complex spatial economic systems: migration, industrial location and regional asymmetries", *Spatial Economic Analysis*, 16(1), pp.1-8.

Capello, R., (2016) *Regional Economics*, New York, Routledge.

Cerina, F., Zhu, Z., Chessa, A. and Riccaboni, M., (2015) "World input-output network", *PLOS ONE*, 10(7), e0134025.

Constantin, D. L., (2021) "Addressing spatial justice at lower territorial levels. some insights from the central and east European countries' perspective", *Regional Science Inquiry*, 13(2), pp.315-326.

Costa, S., Sallusti, F. and Vicarelli, C., (2022) "Trade networks and shock transmission capacity: a new taxonomy of Italian industries", *Journal of Industrial and Business Economics*, 49(1), pp.133-153.

Daley W.M. (1997), *Regional Multipliers: A User Handbook for the Regional Input–Output Modelling System (RIMS II)*, U.S. Department of Commerce, Economics and Statistics Administration, U.S.

Government Printing Office, Washington, DC 20402, Third edn, available online, www.bea.gov/scb/pdf/regional/perinc/meth/rims2.pdf.

De Dominicis, L., (2014) "Inequality and growth in European regions: Towards a place-based approach", *Spatial Economic Analysis*, 9(2), pp.120-141.

del Rio-Chanona, R.M., Grujic, J., Jeldtoft Jensen, H., (2017) "Trends of the world input and output network of global trade", *PLOS ONE*, 12(1), e0170817.

Domar, E., (1946) "Capital Expansion, Rate of Growth, and Employment", *Econometrica*, 14(2), pp.137–147.

Dominguez, A., Mendez, C., (2019) "Industrial productivity divergence and input-output network structures: Evidence from Japan 1973–2012", *Economies*, 7(2), 52.

Dominguez, A., Santos-Marquez, F. and Mendez, C., (2021) "Sectorial productivity convergence, input-output structure and network communities in Japan", *Structural Change and Economic Dynamics*, 59, pp.582-599.

Elhorst, J. P., (2010) "Applied spatial econometrics: raising the bar", *Spatial economic analysis*, 5(1), pp.9-28.

Faggian A. and Biagi B. (2003), 'Measuring Regional Multipliers: A Comparison Between Different Methodologies in the Case of the Italian Regions', *Scienze Regionali – Italian Journal of Regional Science*, vol. 2, no. 1, pp. 33–58.

Garcia-Muniz, A. S., & Ramos-Carvajal, C., (2015) "Input-output linkages and network contagion in Greece: Demand and supply view", *Applied Econometrics and International Development*, 15(2), pp.35-52.

Giovanni, G., Francesco, T., (2008) "Spillover diffusion and regional convergence: a gravity approach", *Regional Science Inquiry*, 71.

Google Maps, (2019) *Google Mapping Services*, available at the URL: www.google.gr/maps?hl=el [last accessed: 29-8-2019]

Goula, M., Ladias, Christos, Ap., Gioti-Papadaki, O., Hasanagas, N., (2015) "The spatial dimension of environment-related attitudes: does urban or rural origin matter?", *Regional Science Inquiry*, 7(2), pp.115-129.

Hamm, R., (2010) "Some Supplementary Regional Economic Effects of a Premier League Soccer Club: Theoretical and empirical Considerations beyond Regional Multiplier Analysis", *Regional Science Inquiry*, 2(1), pp.53-62.

Harrod, R. F., (1939) "An Essay in Dynamic Theory", *The Economic Journal*, 49 (193), pp.14–33.

Hellenic Statistical Service – ELSTAT (2011) "Results of the Census of Population-Habitat 2011 referring to the permanent population of Greece", *Newspaper of Government* (ΦΕΚ), Second Issue (T-B), Number 3465, 28 December 2012.

Karras, G., (2010) "Regional economic growth and convergence, 1950-2007: Some empirical evidence", *Regional Science Inquiry*, 2(1), pp.11-24.

Keynes, J. M., (1937) "The general theory of employment", *The quarterly journal of economics*, 51(2), pp.209-223.

Kuznets, S., (1967) *Modern Economic Growth*, Connecticut: Yale University Press.

Ladias, Christos, Ap., Hasanagas, N., Papadopoulou, E., (2011) "Conceptualising 'macro-regions': Viewpoints and tools beyond NUTS classification", *Studies in Agricultural Economics*, 113(1316-2016-102776), pp.138-144.

Lagos, D., (2007) *Theories of Regional Economic Development*, Athens, Kritiki Publications [in Greek].

Leontief, W., (1966) *Input-Output Economics*. New York: Oxford University Press.

López-Bazo, E., Monastiriotis, V., Ramos, R., (2014) "Spatial inequalities and economic growth", *Spatial Economic Analysis*, 9(2), pp.113-119.

Miller, R., Blair P. (2009), *Input–Output Analysis, Foundations and Extensions*, 2nd edition. Cambridge: Cambridge University Press.

Mirzaei, O., Natcher, D. C., Micheels, E. T., (2021) "A spatial model of investment behaviour for First Nation governments", *Spatial Economic Analysis*, 16(4), pp.530-549.

Nijkamp, P., (2011) "The role of evaluation in supporting a human sustainable development: a cosmonomic perspective", *Regional Science Inquiry Journal*, 3(1), pp.13-22.

North, D. C., (1955) "Location theory and regional economic growth", *Journal of political economy*, 63(3), pp.243-258.

Polyzos, S., (2006) "Public investments and Regional Development: The role of Regional Multipliers", *International Journal of Sustainable Planning and Development*, 1(3), pp.1-16.

Polyzos, S., (2015) *Urban development*, Athens, Kritiki Publications [in Greek].

Polyzos, S., (2019) *Regional Development.* Athens: Kritiki Publications (In Greek).

Polyzos, S., Sofios, S., (2008) "Regional multipliers, Inequalities and Planning in Greece", *South Eastern Europe Journal of Economics*, 6(1), pp.75-100.

Polyzos, S., Tsiotas, D., (2020) "The contribution of transport infrastructures to the economic and regional development: a review of the conceptual framework", *Theoretical and Empirical Researches in Urban Management*, 15(1), pp.5-23.

Rodrigue, J. P., Comtois, C., Slack, B., (2013) *The Geography of Transport Systems*, New York, Routledge Publications.

Steele D.B. (1969), 'Regional Multipliers in Great Britain', *Oxford Economic Papers*, New Series, vol. 21, no. 2, pp. 268–292.

Tiebout C. (1960), "The Community Income Multiplier: A Case Study", in R. Pfouts (ed.), *The Techniques of Urban Economic Analysis* , Chandler-Davis, London

Tiebout, C. M., (1956) "Exports and regional economic growth", *Journal of political economy*, 64(2), pp.160-164.

Trusova, N. V., Prystemskyi, O. S., Hryvkivska, O. V., Sakun, A. Z., Kyrylov, Y. Y., (2021) "Modeling of system factors of financial security of agricultural enterprises of Ukraine", *Regional Science Inquiry*, 13(1), pp.169-182.

Tsiotas, D., (2019) "Detecting different topologies immanent in scale-free networks with the same degree distribution", *Proceedings of the National Academy of Sciences* (PNAS), 116(14), pp.6701-6706.

Tsiotas, D., (2021) "Drawing indicators of economic performance from network topology: the case of the interregional road transportation in Greece", *Research in Transportation Economics*, 90: 101004.

Tsiotas, D., Aspridis, G., Gavardinas, I., Sdrolias, L., Skodova – Parmova, D., (2018) "Gravity modeling in Social Science: the case of the commuting phenomenon in Greece", *Evolutionary and Institutional Economics Review*, 16, pp.139-158.

Tsiotas, D., Axelis, N., Polyzos, S., (2022) "Detecting city-dipoles in Greece based on intercity commuting", *Regional Science Inquiry*, 14(1), pp.11-30.

Tsiotas, D., Ducruet, C., (2021) "Measuring the effect of distance on the network topology of the Global Container Shipping Network", *Scientific Reports*, 11: 21250.

Tsiotas, D., Polyzos, S., (2018) "The complexity in the study of spatial networks: an epistemological approach", *Networks and Spatial Economics*, 18(1), pp.1–32.

Tsiotas, D., Polyzos, S., (2021) "Making the Web-Science operational for interregional commuting analysis: Evidence from Greece", *Journal of the Knowledge Economy*, 12, pp.567-577.

Tsiotas, D., Tselios, V., (2023) "Measuring the Interaction between the Interregional Accessibility and the Geography of Institutions: the case of Greece", In Storti, L., Urso, G., Reid, N., (Eds.) *Economies, institutions, and territories: Dissecting Nexuses in a Changing World (The Dynamics of Economic Space)*, New York, Routledge (doi: 10.4324/9781003191049), pp.269-294.

Xanthos, G., Ladias, Christos, Ap., Genitsaropoulos, C., (2012) "Regional Inequalities In Greece A Proposition For Their Depiction", *Regional Science Inquiry*, 4(2), pp.191-196.

Zeenat Fouzia, S., Mu, J., Chen, Y., (2020) "Local labour market impacts of climate-related disasters: a demand-and-supply analysis", *Spatial Economic Analysis*, 15(3), pp.336-352.

## Appendix

The income multipliers *m*-function in MATLAB
(it can be used whether copied and pasted to a MATLAB's function format)

```
-----------------------------------------------------------------------------------------
function [ T DSM DVM ] = multipliers_multireg( W, dD, impC, MPC)
%MULTIPLIERS_MULTIREG_SYSTEM This function computes the regional income multipliers
and the income changes due to changes in the demand captured in a multiregional
system.
%
% Ref (1). Tsiotas, D., (2022) "A network-based algorithm for computing keynesian
income multipliers in multiregional systems", Regional Science Inquiry, 14(2).
% Ref (2). Polyzos, S., (2019) Regional Development, Athens: Kritiki Publications [in
Greek]
%
%   INPUTS
%   W:      Adjacency matrix, a binary matrix including ones in locations ij if
%           regions Ri and Rj are connected. If a weighted connectivity matrix is
%           imported, it is converted to its adjacency
%   dD:     Change in demand, vector including the change(s) in demand
%           dD(i) = D1(i)-Do(i) in one (or more, i=1,2,... =<n) region(s) Ri.
%           that a prefecture satisfies through imports.
%   impC:   Imports coefficients, vector with the proportions of demand
%           that a prefecture satisfies through imports. If not entered,
%           is randomized, impC = [rand() rand() ... rand()]. Note: it
%           should be assigned in normalized form [0,1].
%   MPC:    Vector of Marginal Propensity to Consume per region
%
%   OUTPUTS
%   T:      Tabulation matrix with the results [1:Region 2:ΔD 3:MPC 4:m 5:ΔP],
%   where
%   m:      Income multipliers vector, including the income multipliers of regions due
to the increase of demand
%   ΔD:     Change in Demand in all regions
%   ΔP:     Income increase vector, including the income increase in regions due
%           to the increase of demand
%   DSM:    Demand Shares Matrix, matrix including the proportions (shares) at
%           which a region satisfies its demand through intrinsic production (at
%           the main diagonal) and imports.
%   DVM:    Demand Shares Matrix, matrix including the
%           volume of intrinsic satisfaction of demand and through imports.
%
%   Developed by Dimitrios Tsiotas, Ph.D., Assistant Professor, on 26/6/2022

tic
W(isnan(W))=0;  % removes NaNs from connectivity matrix
% Check (#0) of vertically aligned arguments
sdD=size(dD);
simpC=size(impC);
sMPC=size(MPC);
if sdD(1)<sdD(2)
    dD=dD';
end
if simpC(1)<simpC(2)
    impC=impC';
end
if sMPC(1)<sMPC(2)
    MPC=MPC';
end
% end of Check#0
% Check (#1) of square structure
s=size(W);
if s(1)~=s(2)
    n=min(s(1),s(2));
    W=W(1:n,1:n);         % keeps only the nxn table
else
    n=s(1);
end
% end of Check#1
% Conversion (#2) of non-symmetric adjacency
Adj_temp=W>0;
if sum(sum(W))~=sum(sum(Adj_temp))
   W=W-W';
   W(W<0)=0;   % removes negative entries
   Adj=eye(n)+W; % fills the main diagonal with ones
   Adj(Adj>1)=1;    % non-symmetric adjacency (to make the structure reasonable for
one-product trade balance)
end
if sum(sum(W))==sum(sum(Adj_temp))
```

```
   Adj=W;
end
% end of conversion #2
% Check (#3) of MPC argument
if nargin<4
    MCP=ones(1,n); % MPC is set to one for all regions
end
% end of Check#3
% Check (#4) of impC argument
if nargin<3
    impC=rand(1,n);
end
% end of Check#4
% Check (#5) of dD argument
if nargin<2
    temp=rand(1,n);
    tM=max(temp);
    dD=(temp==tM);
    dD=dD.*temp;
end
% end of Check#5
% Check (#6) coefficient form of impC (should be <=1)
if impC>1
    if min(impC)<=100;
        impC=impC/100;
    else
    cm=min(impC);
    cM=max(impC);
    for i=1:n;
    impC(i)=impC(i)/(cM-cm); % normalizes values
    end
    end
end
% end of Check#6
intrC=1-impC;        %finds the complementary coefficients, of intrinsic production
AC=Adj-eye(n);      % keep from Adj only connections with seller regions
sellers=sum(AC)’;   % finds the number of seller regions per node
so=(sellers==0); % finds regions that have no sellers due to the network structure
% Check#7, regions with no sellers are set impC=1
intrC=intrC.*(abs(1-so));
intrC=intrC+so;      % sets fully intrinsic regions with no sellers
% end of Check#7
AintrC=diag(intrC); % intrC set to diagonal form
imporpo=impC./sellers;   % proportions satisfied by imports per region
imporpo(isinf(imporpo))=0;   % replaces inf with zeros
% Loop#8, calculation of demand coefficients matrix
for i=1:n
    DSM(:,i)=AC(:,i)*imporpo(i);
end
DSM=DSM+AintrC;
% end of Loop#8
DVM=eye(n);
% Loop#9, calculation of demand volumes
for i=1:n
    DVM(:,i)=DSM(:,i)*dD(i);
end
dvms=zeros(n,1);
% end of Loop#9
% Loop#10, calculation of secondary demand volumes
for j=1:n
    if sum(sum(DVM)==0)<n
dvms=dvms+(dvms==0).*sum(DVM,2); % finds the shared demand volume
dv2=(abs(1-dD>0)); % index of secondary demand, finds regions who secondarily
increased their demand
dvm2 = dvms.*dv2;    % finds the shared demand volumes
for i=1:n
    DVM2(:,i)=DSM(:,i)*dvm2(i);
end
DVM=DVM+DVM2.*(DVM==0);
end
end
% end of Loop#10
indemandchange=sum(DVM.*eye(n))’; % the demand that each regions satisfies
% Loop#11, calculation of regional multipliers
for i=1:n
m(i)=1/(1-MPC(i));
end
m=m’;
% end of Loop#11
```

```
T=[(1:n)’ indemandchange MPC m indemandchange.*m];
toc
display(‘**************************************************’)
display(‘LABELS in Tabulation Matrix:’)
display(‘1st column: Region’)
display(‘2nd column: ΔD - Change in Demand’)
display(‘3rd column: MPC - Marginal Propensity to Consume’)
display(‘4th column: m - Regional Income Multiplier’)
display(‘5th column: ΔP - Change in Product/Income’)
display(‘**************************************************’)

end
-------------------------------------------------------------------------------------
```