OUTPUT, GROWTH, AND CONVERGENCE IN A GREATIVE REGION: AN ANALYSIS OF SOME MEASUREMENT ISSUES [1]

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Abstract
We study some measurement issues that arise when analyzing the long run behavior of the jth creative region's time t log output per creative class member $(y_j(t))$ when this region is part of an aggregate economy of $j = 1, \ldots, N$ creative regions. We focus first (second) on absolute (relative) convergence. In the absolute (relative) convergence case, the N creative regions are similar (dissimilar) in that they all have the same (different) balanced growth path (BGP) level of log output per creative class member denoted by y^{BGP} (y_j^{BGP}). In the absolute convergence case, we analyze how to estimate the speed of convergence parameter (σ) and then discuss the relationship between the variance of $y_j(t)$ and that of $y_j(0)$. In the relative convergence case, we study the error associated with estimating σ using the methodology of the absolute convergence case. Finally, suppose $y_j^{BGP} = a + bX_j$ where X_j is an explanatory variable such as creative capital and a and b are positive constants. Here, we study how to estimate b from our knowledge of the coefficients of a related cross-region growth regression.
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1. Introduction

1.1. Preliminaries and objective

In the regional science and urban economics literatures, much has now been written about the notion of creativity in general and about creative regions in particular. This state of affairs is primarily due to the work of Richard Florida[2] who has popularized the twin notions of the *creative class* and *creative capital*. Florida (2002, p. 68) helpfully explains that the creative class "consists of people who add economic value through their creativity." This class is made up of professionals such as doctors, lawyers, scientists, engineers, university professors, and, notably, bohemians such as artists, musicians, and sculptors. From the standpoint of regional economic growth, these people are significant because they possess creative capital which is the "intrinsically human ability to create new ideas, new technologies, new business models, new cultural forms, and whole new industries that really [matter]" (Florida, 2005a, p. 32).

As noted by Florida (2005b), the creative class deserves to be studied in detail because this group gives rise to ideas, information, and technology, outputs that collectively promote a region's economic growth. Therefore, in this era of globalization, cities and regions that want to be successful need to do all they can to attract and retain members of the creative class because this class is markedly responsible not only for a region's economic growth but, more generally, for its welfare as well.

In their own ways, several researchers have now pointed to a connection between a region's output or income and the activities of this region's creative class. For instance, Markusen *et al.* (2008) focus on the Boston area and point to the importance of the size of the

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[2] See Florida (2002, 2005a) and Florida *et al.* (2008).

creative economy in formulating sound cultural policy and in forming creative regions. Waitt and Gibson (2009) concentrate on Wollongong in Australia and show that if one is to comprehend the surprising ways in which urban regeneration occurs then it is necessary to first conceptualize the pertinent creative economy qualitatively in place. Currid-Halkett and Stolarick (2013) ask whether the creative class generates economic growth and show that although creativity matters in general, different sub-groups within the creative class have differential impacts on unemployment and growth.

Lee (2014) focuses on different regions in the United Kingdom and shows that creative industries drive both wage and employment growth in other sectors. Correia and Costa (2014) analyze the connection between creativity and economic growth in European Union member states by evaluating the relative performance of alternate indices of creativity. Porter and Batabyal (2016) study a model with two regions and show how to decompose the difference in the logarithm of the output of the knowledge good per raw creative capital unit between the two regions into the contributions of education and that of all other factors.

The papers discussed in the preceding two paragraphs have certainly advanced our understanding of the link between the activities of the creative class in a region and economic growth in this same region. However, to the best of our knowledge, the existing literature has *not* studied some measurement issues that arise when analyzing the long run behavior of the jth creative region's time t log output per creative class member $(y_j(t))$ when this region is part of an aggregate economy of $j = 1, \ldots, N$ creative regions. These measurement issues arise because of our interest in determining the *speed* at which $y_j(t)$ converges to its long run value and from the fact that there are two *different* ways in which we can study the question of long run output convergence in a creative region. Given this lacuna in the literature, our objective in the present paper is to shed light on these measurement issues. We now discuss these measurement issues in greater detail.

1.2. The measurement issues

Consider a stylized, infinite horizon region j that is part of an aggregate economy of $j = 1, \ldots, N$ regions. These N regions are all creative in the sense of Richard Florida. This means that they all possess a vibrant creative class and that the individual members of the creative class possess creative capital. Hence, we can think of each creative class member as a creative capital unit. In the remainder of this paper, we shall treat these creative capital units as essential inputs into a production process that results in the output of a knowledge good such as a smartphone. Let $y_j(t)$ denote the log output of this knowledge good per creative capital unit at any time t.

We first consider the case of *absolute* convergence. In this case, the long run or balanced growth path (BGP) level of log output of the knowledge good per creative capital unit in the jth region is y^{BGP} and this value is the *same* in all the N regions under consideration. In addition, the evolution of $y_j(t)$ over time is described by the differential equation

$$\frac{dy_j(t)}{dt} = \dot{y}_j(t) = -\sigma\{y_j(t) - y^{BGP}\}, \tag{1}$$

where σ is the speed of convergence parameter. There are now two measurement issues to contend with. First, we want to estimate the speed of convergence parameter σ as accurately as possible. Second, we would like to characterize the relationship between the variance of $y_j(t)$ and that of $y_j(0)$.

Moving on to the case of *relative* convergence, the BGP level of log output of the knowledge good per creative capital unit in the jth region is y_j^{BGP} and this value is *different* in all the N regions under consideration. Moreover, the evolution of $y_j(t)$ over time is now described by the differential equation[3]

$$\frac{dy_j(t)}{dt} = \dot{y}_j(t) = -\sigma\left\{y_j(t) - y_j^{BGP}\right\}. \tag{2}$$

The two measurement issues that we study in this case are the following. First, we want to delineate the nature of the error stemming from our estimation of σ using the methodology for the absolute convergence case. Second, suppose that the BGP value of $y_j(t)$ or y_j^{BGP} can be described by the linear relationship $y_j^{BGP} = a + bX_j$ where X_j is an explanatory variable such as creative capital and a and b are positive constants. Here, we want to estimate b from our knowledge of the coefficients of a related cross-region growth regression that is described in greater detail in section 3.3 below.

With this introductory discussion of the measurement issues out of the way, we can now say that the remainder of this note is organized as follows. Sections 2 and 3, respectively, focus on the case of absolute and relative convergence. Specifically, section 2.1 derives a linear relationship satisfied by $y_j(t)$. Section 2.2 provides an estimate of σ. Section 2.3 specifies the relationship between the variance of $y_j(t)$ and that of $y_j(0)$. Section 3.1 derives a new linear relationship satisfied by $y_j(t)$. Section 3.2 stipulates the error involved in estimating σ using the methodology of section 2.2. Section 3.3 discusses the estimation of the coefficient b from our knowledge of the coefficients of a particular cross-region growth regression. Section 4 concludes and then offers two suggestions for extending the research delineated in this paper.

2. Absolute Convergence

2.1. The linear relationship

Our task now is to describe $y_j(t)$ as a linear function of the explanatory variables $y_j(0), y^{BGP}, \sigma,$ and time t. Given equation (1) and the fact that y^{BGP} is a constant, it is clear that the time derivative of $y_j(t)$ is the same as the time derivative of $y_j(t) - y^{BGP}$. Therefore, we can rewrite equation (1) as

$$\frac{d\{y_j(t) - y^{BGP}\}}{dt} = -\sigma\{y_j(t) - y^{BGP}\}. \tag{3}$$

Inspecting equation (3), it is straightforward to confirm that the quantity $y_j(t) - y^{BGP}$ is growing over time at rate $-\sigma$. This finding allows us to infer that

$$y_j(t) - y^{BGP} = e^{-\sigma t}\{y_j(0) - y^{BGP}\}. \tag{4}$$

Isolating $y_j(t)$ on the left-hand-side (LHS), equation (4) can be rewritten as

$$y_j(t) = (1 - e^{-\sigma t})y^{BGP} + e^{-\sigma t}y_j(0). \tag{5}$$

[3] In the macroeconomics literature, the notions of absolute and relative convergence are sometimes referred to as unconditional and conditional convergence. See Romer (2012, pp. 178-188) for additional details.

Equation (5) gives us the linear relationship satisfied by $y_j(t)$ that we seek. We now use this particular relationship to show how we might provide an estimate of the speed of convergence parameter σ.

2.2. Estimating the speed of convergence

As a prelude to estimating σ, suppose that the true relationship satisfied by $y_j(t)$ is the right-hand-side (RHS) of equation (5) plus a random component $(z_j(t))$ that has zero mean and is uncorrelated with $y_j(0)$. Then, we have

$$y_j(t) = (1 - e^{-\sigma t})y^{BGP} + e^{-\sigma t}y_j(0) + z_j(t). \quad (6)$$

Now, in the process of estimating σ, suppose we run a cross-region growth regression of the form

$$y_j(t) - y_j(0) = \alpha + \beta y_j(0) + \epsilon_j, \quad (7)$$

where α and β are positive constants and the error term ϵ_j has the standard statistical properties.

To provide an estimate of σ, we shall first have to determine the relationship between β and σ. We know from standard econometrics---see Kelejian and Oates (1981, p. 56)---that the coefficient β in equation (7) can be expressed as

$$\beta = \frac{Cov\{y_j(t) - y_j(0), y_j(0)\}}{Var\{y_j(0)\}}, \quad (8)$$

where $Cov(\cdot,\cdot)$ is the covariance function and $Var(\cdot)$ is the variance function. Now, using standard formulae for the covariance and the variance of random variables---see Ross (2003, pp. 53-61)---we can simplify the RHS of equation (8). This tells us that

$$\beta = \frac{Cov\{y_j(t), y_j(0)\}}{Var\{y_j(0)\}} - 1. \quad (9)$$

Using equation (6), the numerator in the fraction on the RHS of equation (9) can be simplified to

$$Cov\{y_j(t), y_j(0)\} = Cov\{(1 - e^{-\sigma t})y^{BGP} + e^{-\sigma t}y_j(0) + z_j(t), y_j(0)\}. \quad (10)$$

Now recall that y^{BGP} is a constant and that $y_j(0)$ and $z_j(t)$ are assumed to be uncorrelated. Using these two pieces of information, we can simplify the RHS of equation (10). This yields $Cov\{y_j(t), y_j(0)\} = e^{-\sigma t}Var\{y_j(0)\}$. Substituting this last finding in equation (9), we get

$$\beta = \frac{e^{-\sigma t}Var\{y_j(0)\}}{Var\{y_j(0)\}} - 1 = e^{-\sigma t} - 1. \quad (11)$$

Simplifying the RHS of equation (11) by taking the natural logarithm, we get

$$\sigma = -\frac{\ln(1+\beta)}{t}. \tag{12}$$

Equation (12) gives us the answer to a measurement issue in the absolute convergence case that we are studying presently. Specifically, we see that once we run the cross-region growth regression described in equation (7), we have an estimate of β. Given this estimate of β, we can then use equation (12) to provide us with an accurate estimate of the speed of convergence parameter σ. From equations (7) and (12), we see that as the marginal impact of a change in the initial time log output of the knowledge good per creative capital unit on the corresponding time t log output *rises* or, equivalently, as *rises*, the speed with which the jth creative region's time t log output converges to its BGP level *falls*. We now proceed to study the relationship between the variance of $y_j(t)$ and that of $y_j(0)$.

2.3. Relationship between two variances

From equation (6), we can derive the variance of $y_j(t)$. We get

$$Var\{y_j(t)\} = e^{-2\sigma t}Var\{y_j(0)\} + Var\{z_j(t)\}. \tag{13}$$

Now first consider the case where β in equation (7) is negative. This means that creative regions that are wealthier on average grow *less* than creative regions that are poorer on average. From equation (12), we see that $\beta < 0$ implies that the speed of convergence parameter or $\sigma > 0$. What does this mean for the variances of $y_j(t)$ and $y_j(0)$? We would like to have a situation where $Var\{y_j(t)\} < Var\{y_j(0)\}$. This would mean that the cross-region variance of log output per creative capital unit is *declining*. However, this inequality between the two variances does *not* have to hold. This is because of the variance of the random component of log output denoted by $Var\{z_j(t)\}$ in equation (13). In other words, when β is negative or when σ is positive we have a scenario in which the dispersion of log output is attenuated. However, this attenuation *can* be counteracted by the random component of log output which tends to *increase* the dispersion of log output.

Next, let us consider what happens when β in equation (7) is positive. This condition tells us that creative regions that are wealthier on average grow *more* than creative regions that are poorer on average. Since this case is the opposite of the case discussed in the preceding paragraph, we now have $\sigma < 0$. In addition, we also have $Var\{y_j(t)\} > Var\{y_j(0)\}$. Put differently, the upshot of $\beta > 0$ or $\sigma < 0$ is to *increase* the dispersion of log output and this effect tends to *strengthen* the effect of the random component in equation (13) which also tends to *increase* the dispersion of log output. This completes our discussion of measurement issues in the absolute convergence case. We now analyze some measurement issues in the case where convergence is relative.

3. <u>Relative Convergence</u>

3.1. The linear relationship

Our first task is to derive a linear relationship between $y_j(t)$ and the explanatory variables $y_j(0), X_j, \sigma,$ and time t. Now, note that given equation (2) and the fact that $y^{BGP} = a + bX_j$ is constant or time invariant, an analysis similar to that employed in section 2.1---see equation (5) in particular---tells us that

$$y_j(t) = (1 - e^{-\sigma t})(a + bX_j) + e^{-\sigma t}y_j(0). \tag{14}$$

Equation (14) gives us the linear relationship satisfied by $y_j(t)$ that we seek. We now use this relationship to delineate the error involved in estimating the speed of convergence parameter σ using the methodology of section 2.2.

3.2. Deducing the speed of convergence

Suppose that $y_j(0) = y_j^{BGP} + z_j$, where z_j is a mean zero random component as in section 2.2. In addition, $y_j(t)$ is given by equation (14) plus a mean zero random term ε_j and we assume that X_j, z_j, and ε_j are uncorrelated with each other. Suppose we run a cross-region growth regression of the form

$$y_j(t) - y_j(0) = \alpha + \beta y_j(0) + \epsilon_j, \tag{15}$$

where the error component ϵ_j has, as in section 2.2, the standard statistical properties. What we want to know now is the magnitude of the error we would make if we deduce the value of σ from our estimate of β in equation (11).

To determine this magnitude and for comparative purposes, let us first compute the estimate of β implied by the model of this section in which convergence is relative and not absolute. Using equation (15) and the methodology employed in section 2.2, once again, we get

$$\beta = \frac{Cov\{y_j(t), y_j(0)\}}{Var\{y_j(0)\}} - 1. \tag{16}$$

The numerator in the ratio on the RHS of equation (16) can be simplified because we have

$$Cov\{y_j(t), y_j(0)\} = (1 - e^{-\sigma t})Cov\{y_j^{BGP}, y_j(0)\} + e^{-\sigma t}Var\{y_j(0)\}. \tag{17}$$

Because $y_j(0) = y_j^{BGP} + z_j = a + bX_j + z_j$, we can tell that

$$Var\{y_j(0)\} = b^2 Var\{X_j\} + Var\{z_j\} \tag{18}$$

and, because X_j and z_j are assumed to be uncorrelated, we also get

$$Cov\{y_j^{BGP}, y_j(0)\} = Cov\{a + bX_j, a + bX_j + z_j\} = b^2 Var\{X_j\}. \tag{19}$$

Now, substituting equations (18) and (19) into equation (17) and then simplifying, we get

$$Cov\{y_j(t), y_j(0)\} = b^2 Var\{X_j\} + e^{-\sigma t} Var\{z_j\}. \tag{20}$$

Using equations (18) and (20) to simplify equation (16), we get

$$\beta = \frac{-(1 - e^{-\sigma t})Var\{z_j\}}{b^2 Var\{X_j\} + Var\{z_j\}}. \tag{21}$$

We are now in a position to use equation (21) to solve for the speed of convergence parameter σ explicitly. Isolating the term $e^{-\sigma t}$ and then taking the natural logarithm of both sides of the resulting expression gives us

$$\sigma = -\frac{\ln\left[\left\{\frac{b^2 Var\{X_j\} + Var\{z_j\}}{Var\{z_j\}}\right\}\beta + 1\right]}{t} \qquad (22)$$

Inspecting equation (22), observe that the ratio in the curly brackets on the RHS or $\left(b^2 Var\{X_j\} + Var\{z_j\}\right) / \left(Var\{z_j\}\right) > 1.$

This last piece of information and some thought together tell us that if we were to use equation (11) or (12) from section 2 on absolute convergence to deduce σ in the case of relative convergence then this action would result in our estimating a value for this speed of convergence parameter that is *too small* in absolute value. In other words, as long as $\sigma > 0,$ using the section 2 methodology to estimate σ would result in an *underestimate* of the true speed of convergence. We now proceed to our last task in this paper and that is to estimate the coefficient b in the relationship $y_j^{BGP} = a + bX_j$ from our knowledge of the coefficients of a particular cross-region growth regression.

3.3. Estimating the coefficient of a BGP relationship

Suppose we run a cross-region growth regression of the form

$$y_j(t) - y_j(0) = \alpha + \beta y_j(0) + \delta X_j + \epsilon_j, \qquad (23)$$

where the error component ϵ_j has the standard statistical properties. Our specific goal in this section is to come up with an estimate of the coefficient b from our knowledge of $\alpha, \beta,$ and $\delta.$ Using equation (14), the fact that $y_j^{BGP} = a + bX_j,$ and the assumptions in section 3.2, we deduce that

$$y_j(t) = \left(1 - e^{-\sigma t}\right)y_j^{BGP} + e^{-\sigma t}y_j(0) + \varepsilon_j. \qquad (24)$$

Subtracting $y_j(0)$ from the LHS and the RHS of equation (24) and then substituting $y_j(0) = y_j^{BGP} + z_j$ into the resulting equation gives us

$$y_j(t) - y_j(0) = \left(e^{-\sigma t} - 1\right)z_j + \varepsilon_j. \qquad (25)$$

Let $\Delta \equiv \left(e^{-\sigma t} - 1\right).$ Then we note that the cross-region growth regression given by equation (23) is equivalent to projecting the term on a constant, $y_j(0),$ and X_j and it is understood that the random error term ε_j is uncorrelated with the just mentioned RHS variables. We can now rearrange the equation $y_j(0) = a + bX_j + z_j$ to solve for $z_j.$ We get

$$z_j = -a - bX_j + y_j(0). \qquad (26)$$

Multiplying both sides of equation (26) by *Δ* we get

$$\Delta Zj = -\Delta\alpha - \Delta bXj + \Delta yj(0) \qquad (27)$$

Inspecting equations (23) and (27), we see that the estimate of β in (23) provides us with an estimate of Δ in (27) and, similarly, the estimate of δ in (23) provides us with an estimate of

-Δb in (27). With these two pieces of information in hand, we infer that our estimate of b is the negative of the estimate of δ divided by the estimate of β. In symbols, we have

$$b = -\frac{\delta}{\beta} = -\frac{\Delta b}{\Delta}. \quad (28)$$

Equation (28) provides us with the answer to the central question of this section. This also concludes our analysis of output, economic growth, and absolute and relative convergence in a creative region.

4. Conclusions

In this paper, we studied some measurement issues that arose in the context of the long run behavior of the jth creative region's time t log output per creative class member $(y_j(t))$ when this region was part of an aggregate economy of $j = 1, \ldots, N$ creative regions. We concentrated first (second) on absolute (relative) convergence. In the absolute (relative) convergence case, the N creative regions were similar (dissimilar) in that they all had the same (different) BGP level of log output per creative class member denoted by y^{BGP} (y_j^{BGP}). In the absolute convergence case, we analyzed how to estimate the speed of convergence parameter (σ) and then discussed the relationship between the variance of $y_j(t)$ and that of $y_j(0)$. In the relative convergence case, we studied the error associated with estimating σ using the methodology of the absolute convergence case. Finally, we supposed that $y_j^{BGP} = a + bX_j$ where X_j was an explanatory variable such as creative capital and a and b were positive constants. Here, we studied how to estimate b from our knowledge of the coefficients of a related cross-region growth regression.

The analysis in this paper can be extended in a number of different directions. Here are two suggestions for extending the research described here. First, it would be useful to explicitly allow for economic *interactions* between the N creative regions and to then conduct a convergence analysis of the sort carried out in this paper. Second, it would also be instructive to study in greater detail the extent to which differences in the growth rates of, for instance, log output between different creative regions are dependent on their *initial* positions relative to their balanced growth paths (BGPs). Studies that incorporate these aspects of the problem into the analysis will increase our understanding of the nexuses between a creative region's economic interactions and its initial position on the one hand and its log run economic growth prospects on the other.

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