THE CHAOTIC UNEMPLOYMENT RATE GROWTH MODEL: EURO AREA

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Abstract
The unemployment rate in the Euro Area fall to 10.026 per cent in 2016. Among the Member States, the lowest unemployment rates were recorded in Germany and Malta. On the other hand, the highest unemployment rates were observed in Greece and Spain. Unemployment rates have fallen from their postcrisis peaks, but remain high. The basic aims of this paper are: firstly, to provide a relatively simple chaotic unemployment rate growth model that is capable of generating stable equilibria, cycles, or chaos, and secondly, to to analyze the unemployment rate growth stability in the period 1991-2015 in the Euro Area.. This paper confirms stable growth of the unemployment rate in the Euro Area in the observed period.
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1. Introduction

The Euro Area economy continued its gradual recovery in 2016. GDP Growth Rate in the Euro Area averaged 0.37 percent from 1995 until 2016. GDP Growth Rate in the Euro Area stood at 1.661 percent in 2016 (see Fig. 1).

While private consumption continued to expand, supported by higher real disposable incomes as a result of lower energy prices and rising employment, investment growth faltered.

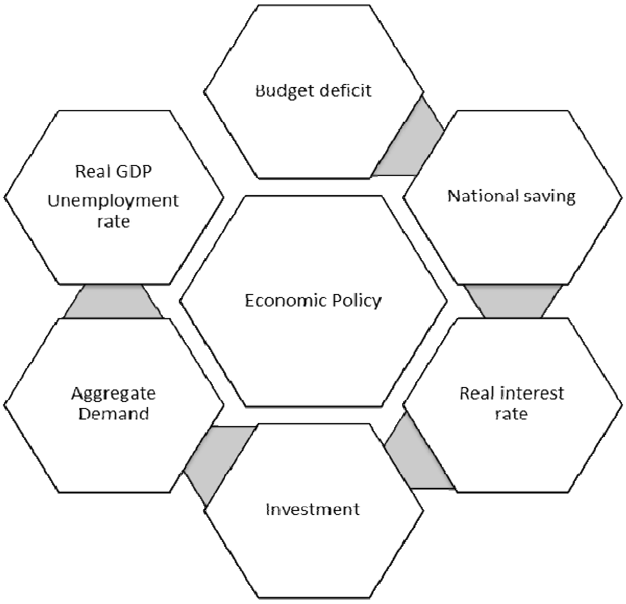
Euro Area inflation has been falling steadily for five years . In 2016, the Euro Area inflation rate was just 0.278 % . Inflation is also picking up in the Euro Area, but more slowly and from from 0.033 percent in 2015, to 0.278 percent in 2016.

In the Euro Area domestic demand is expected to be the main driver with robust private consumption supported by rising real incomes reflecting higher employment. Domestic demand, notably investment, decelerated in some of the larger euro area economies. Domestic demand and investment are still below precrisis levels in some euro area countries. In the Euro Area, GDP and investment have in recent years grown more slowly than projected. On the other hand, employment has grown faster. The productivity slowdown reflects prolonged weak investment.

A modest fiscal expansion and easy monetary policy support economic growth. Persistently low interest rates in the Euro Area are coupled with the expectation of a continuation of the gradual increase of short-term rates in the US. Fiscal policy has been less restrictive in 2016 than in the previous three years. The reduction of the consolidated Euro Area budget deficit has been supported by improved growth. Public consumption is controlled. Countries with high debt burdens should undertake gradual fiscal consolidation.

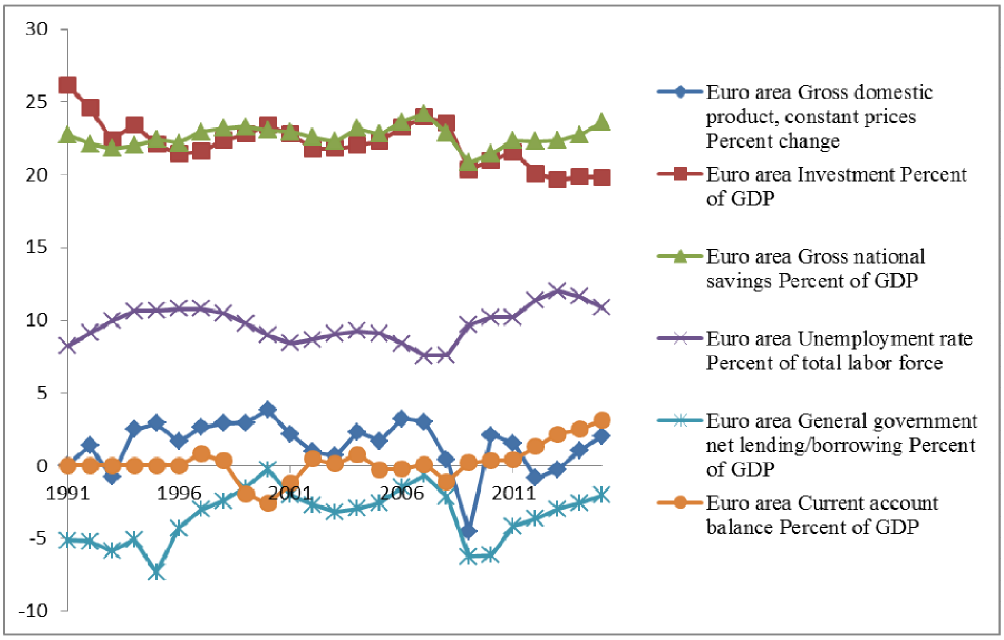
In the short run, the fall in aggregate demand leads to falling output and price level and rising unemployment. What should policymakers do when faced with such a recession? One possibility is to take action to increase aggregate demand. An increase in government spending or an increase in money supply would increase aggregate demand. According to the Phillips curve, when aggregate demand is low , then unemployment is high and inflation is low. On the other hand, Okun's law is a relationship between changes in the unemployment rate and economic growth. In this sense, this relationship predicts that growth slowdowns coincide with rising unemployment. (see Fig 1 and Fig 2.)

Figure 1: Relations between budget deficit, national saving investment and unemployment rate.



Chaos theory states that small changes can result in large differences. Chaotic system is unpredictable. Namely, a slight difference, in the decimal place, resulted in prediction failure. Chaotic systems exhibit a sensitive dependence on initial conditions: seemingly insignificant changes in the initial conditions produce large differences in outcomes. This is very different from stable dynamic systems in which a small change in one variable produces a small and easily quantifiable systematic change. Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981,1982), Day (1982, 1983,1992 , 1997.), Grandmont (1985), Goodwin (1990), Medio (1993,1996), Lorenz (1993), Jablanovic (2011, 2013, 2016), among many others.

Figure 2.GDP, investment, gross national saving, unemployment rate, general government net



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The basic aims of this paper are: firstly, to provide a relatively simple chaotic unemployment rate growth model that is capable of generating stable equilibria, cycles, or chaos, and secondly, to to analyze the unemployment rate growth stability in the period 1991-

2015 in the Euro Area. This paper confirms stable growth of the unemployment rate in the Euro Area in the observed period.

2. The model

The chaotic unemployment growth model is presented by the following equations:

ut - un = - α (Yt - Yn)	α > 0	(1)
un = β ut	β > 0	(2)
Yn = γ Yt	γ > 0	(3)
Y t = Ct + It + Gt + Nxt		(4)
Ct = δ Yt-1 2	0< δ <1	(5)
It = ρ (Yt – Yt-1)	ρ > 1	(6)
Nx, t = n Y t	0< n <1	(7)
Gt = g Yt	0< g <1	(8)

with Y – real output , Yn – the potential output, I – investment , C– consumption, Nx – net exports, G – governmnet spending, u – unemployment rate, un - the natural rate of unemployment, α – the "Okun's coefficient" , δ - the private consumption rate, β and γ – the positive coefficients, n – the net export rate, g- the government expenditure rate, ρ - the accelerator.

(1) shows the Okun' law ; the negative correlation between GDP growth and unemployment has been named "Okun's law." The relationship between contemporaneous changes in economic growth and unemployment is often referred to as "Okun's Law". The parameter α is often called "Okun's coefficient." Okun's relationship connected the level of unemployment to the gap between actual output (Y) and potential output (Yn). Potential output explains how much the economy would produce "under conditions of full employment". (2) shows that the natural rate of unemployment which is known as the non acceleration inflation rate of unemployment (NAIRU) is proportional to the current unemployment rate; (3) shows that the potential output is proportional to the actual output; (4) shows GDP (Y) as the sum of consumption (C), investment (I), government spending (G) and net exports; (5) In this model, the consumption function displays the quadratic relationship between consumption (Ct) and real output of the previous period (Yt-1). Real output is multiplied by the coefficient δ, „the marginal propensity to consume" (MPC) . The MPC coefficient can be between zero and one. (6) As regards investment in period t, it is taken to be the function of the change in real output in the previous period, where ρ stands for the capital –output ratio or accelerator; (7) shows the relation between net export (Nx) and real output (Y); and (8) shows the relation between government spending (G) and real output (Y).

Now, putting (1), (2), (3), (4), (5) , (6), (7), and (8) together we immediately get:

$$u_t = \left[\frac{\rho}{(g+n+\rho-1)}\right]u_{t-1} - \left[\frac{\delta(\beta-1)}{\alpha(1-\gamma)(g+n+\rho-1)}\right]u_{t-1}^{\,2} \qquad (9)$$

This model given by equation (9) is called the logistic model. For most choices of α, β, γ , δ , ρ , g , and n there is no explicit solution for (9). Namely, knowing α, β , γ , δ, ρ , g , and n and measuring u 0 would not suffice to predict u t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems.

Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (9) can lead to very interesting dynamic behavior, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of u t. This difference equation (9) will posses a chaotic region. Two properties of the chaotic solution are important : firstly, given a starting point u0 the solution is highly sensitive to variations of the parameters $\alpha, \beta, \gamma, \delta, \rho, g$, and n; secondly, given the parameters $\alpha, \beta, \gamma, \delta, \rho, g$, and n, the solution is highly sensitive to variations of the initial point u0 . In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

3. The Logistic Equation

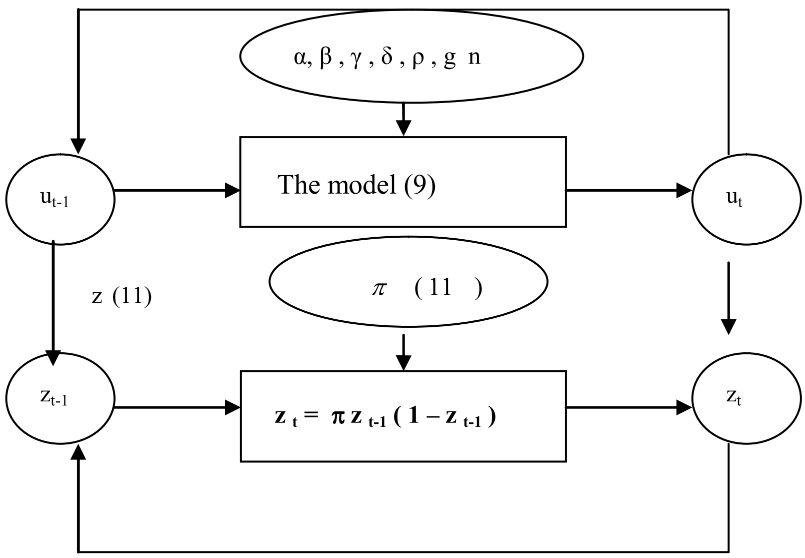
It is possible to show that iteration process for the logistic equation (see Fig. 3.)

$$z_t = \pi z_{t-1} (1 - z_{t-1}), \quad \pi \in [0,4], \quad z_t \in [0,1] \tag{10}$$

is equivalent to the iteration of growth model (9) when we use the identification

$$z_{t-1} = \left[\frac{\delta(\beta-1)}{\alpha\rho(1-\gamma)}\right] u_{t-1} \quad \text{and} \quad \pi = \left[\frac{\rho}{(g+n+\rho-1)}\right] \tag{11}$$

Figure 3: Two quadratic iteratiors running in phase are tightly coupled by the transformations indicated



Using (9) and (11) we obtain:

$$z_t = \left[\frac{\delta(\beta-1)}{\alpha\rho(1-\gamma)}\right] u_t =$$

$$= \left[\frac{\delta(\beta-1)}{\alpha\rho(1-\gamma)}\right]\left\{\left[\frac{\rho}{(g+n+\rho-1)}\right] u_{t-1} - \left[\frac{\delta(\beta-1)}{\alpha(1-\gamma)(g+n+\rho-1)}\right] u_{t-1}^{\ 2}\right\}$$

$$=\left[\frac{\delta\,(\beta-1)}{\alpha\;(1-\gamma)(g+n+\rho-1)}\right]u_{t-1}-\left[\frac{\delta^2\,(\beta-1)^2}{\alpha^2\;\rho\,(1-\gamma)^{\,2}\,(g+n+\rho-1)^2}\right]u_{t-1}{}^2$$

On the other hand, using (10) and (11) we obtain:

$$z_t = \pi\, z_{t-1}\,(\,1 - z_{t-1}\,) =$$

$$=\left\{\frac{\rho}{(g+n+\rho-1)}\right\}\left[\frac{\delta\,(\beta-1)}{\alpha\,\rho\,(1-\gamma)}\right]u_{t-1}\left\{1-\left[\frac{\delta\,(\beta-1)}{\alpha\,\rho\,(1-\gamma)}\right]u_{t-1}\right\}$$

$$=\left[\frac{\delta\,(\beta-1)}{\alpha\;(1-\gamma)(g+n+\rho-1)}\right]u_{t-1}-\left[\frac{\delta^2\,(\beta-1)^2}{\alpha^2\;\rho\,(1-\gamma)^{\,2}\,(g+n+\rho-1)^2}\right]u_{t-1}{}^2$$

Thus we have that iterating (9) is really the same as iterating (10) using (11). It is important because the dynamic properties of the logistic equation (10) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that :

(i) For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;

(ii) For $1 < \pi < 3{,}57$ there exist fixed points the number of which depends on π;

(iii) For $1 < \pi < 2$ all solutions monotnically increase to $z = (\pi - 1) / \pi$;

(iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;

(v) For $3 < \pi < 4$ all solutions will continously fluctuate;

(vi) For $3{,}57 < \pi < 4$ the solution become "chaotic" wihch means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

Important parameter π values " 0, 1 , 1, 2, 3 " are part of the Fibonacci sequence. The Fibonacci Sequence is the series of numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ... There is an interesting pattern: The Fibonacci Sequence is found by adding the two numbers before it together. The is found by adding the two numbers before it (0+1). The 2 is found by adding the two numbers before it (1+1). The 3 is found by adding the two numbers before it (1+2). Namely, each number is the sum of the two numbers before it. If we make squares with those widths, we get a nice spiral. Also, if we take any two successive, important values of parameter π , (" 2, 3 ") , their ratio is very close to the Golden ratio which is approximately 1.618034... The adjacent numbers divided yield the Golden Ratio (e.g. 55/34=1.618). For example 3/2 is 1.5. The golden ratio that has approximate value of 1.618. The golden ratio and the golden rectangle are connected. This is because the ratio of the longer side of a golden rectangle to the shorter side is equal to the golden ratio ($1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + \ldots$) (Jablanovic, 2016., pg. 30)

4. Empirical Evidence

The main aim of this paper is to analyze the unemployment rate stability in the period 1991-2015 in the Euro Area by using the presented logistic model (12):

$$ut \;=\; \pi\, u_{t-1} - \upsilon\, u_{t-1}{}^2 \qquad (12)$$

where u – unemployment rate, $\pi = = \left[\frac{\rho}{(g+n+\rho-1)}\right]$, $\upsilon = \left[\frac{\delta\,(\beta-1)}{\alpha\,(1-\gamma)(g+n+\rho-1)}\right]$

Now, the model (12) is estimated (see Table 1.)

Table 1. The estimated model (12): Euro area , 1991-2015. (R= 0.825 , Variance explained: 68.063%)

	π	**υ**
Estimate	1.198807	1.907698
Std.Err.	0.124436	1.238245
t (22)	9.633944	1.540646
p-level	0 .00000	0.137664

Source: www.imf.org

5. Conclusion

This paper creates the chaotic unemployment rate growth model. For most choices of α, β , γ , δ , ρ , g , and n there is no explicit solution for (9). Namely, knowing α, β , γ , δ , ρ , g , and n and measuring u_0 would not suffice to predict u_t for any point in time, as was previously possible. But even slight deviations from the values of parameters: α, β , γ , δ , ρ , g , n, and initial value of unemployment rate, u_0 show the difficulty of predicting a long-term unemployment rate behavior.

A key hypothesis of this work is based on the idea that the coefficient $\pi = \left[\frac{\rho}{(g+n+\rho-1)}\right]$ plays a crucial role in explaining the local unemployment growth stability, where , n – the net export rate, g- the government expenditure rate, ρ - the accelerator. An estimated value of the coefficient π (1.198007) confirms stable growth of the unemploymet rate in the Euro area in the observed period.

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